VOL. 23 No. 277

WIRELESS ENGINEER

OCTOBER 1946

Editor
W. T. COCKING, M.I.E.E.

Managing Editor
HUGH S. POCOCK, M.I.E.E.

Technical Editor
Prof. G. W. O. HOWE, D.Sc., M.I.E.E.

Editorial Advisory Board.—F. M. COLEBROOK, B.Sc., A.C.G.I. (National Physical Laboratory), L. W. HAYES, O.B.E., M.I.E.E. (British Broadcasting Corporation), Professor E. B. MOULLIN, Sc.D., M.I.E.E., A. H. MUMFORD, O.B.E., B.Sc. (Eng.), M.I.E.E. (G.P.O. Engineering Department), R. L. SMITH-ROSE, D.Sc., Ph.D., M.I.E.E. (National Physical Laboratory).

EDITORIAL

Is Rotation Relative or Absolute?

IN going over some letters of 1935 we came across one referring to an article on "The Magnetic Field" in *Electrical Review*. The writer said: "With most of what you say I am in full agreement, but I am not happy about . . . where you say that the 'rotation is not relative to anything else, not even to the observer.' Surely, the point is that the rotation is *relative to the observer*."

This raises a very interesting point. A cylinder rotating about its axis is shown in Fig. 1. If the observer at P be at rest, *i.e.* relatively to the centre of the cylinder, he will observe that the point A has a velocity v in one direction, whereas the point B has an equal velocity v in the opposite direction. He will conclude that the cylinder as rotating with an angular velocity $\omega = 2v/AB$. One may feel tempted to regard this rotation as only relative to the stationary observer, but this is not so, for let us now assume that the observer is also rotating with the cylinder so that he is always close to the point A on the cylinder. What will he now report concerning the movement of the cylinder? He will observe that the point A is at rest but that the opposite point B is moving with a velocity $2v$, and he will therefore conclude that the cylinder is rotating with the same angular velocity as before, viz. $2v/AB$. To rotate about the centre the observer P must be attached to some arm, or subjected to some mechanical restraint, otherwise he would shoot off at a tangent.* The same result would, of course, be obtained by an observer on the cylinder, say at A. The cylinder is not at rest relatively to an observer on the cylinder, as he would soon discover as the speed was increased. Whatever may be the movement of the observer, whether linear or rotational, he observes the same angular velocity of the cylinder. If the rotation is the same relative to everything in the universe, it is surely absolute. If the cylinder is surrounded by another cylinder and they both rotate with the same angular velocity, one is again tempted to say that one is not rotating relatively to the other, but this is surely a dangerous use of the word "relatively." An observer on either cylinder would report that the other one was rotating, and at the

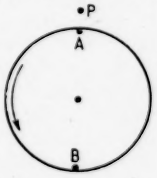


Fig. 1.

* It has been suggested to us that the observer rotating with the cylinder might keep his eyes fixed on the axis and conclude that the cylinder was not rotating at all, but that the rest of the universe was rotating around it. The force which he would experience would be a mystery to him. In the above we have ruled out such an observer and assumed one who did not rotate about his own axis, but kept his eyes fixed on some distant point. There is some ambiguity in describing the motion of such an observer for, although every part of him is rotating about the axis of the cylinder, he himself is not rotating, since he is always looking in the same direction.

same angular velocity as the one on which he was situated. The two cylinders are rotating absolutely at the same angular velocity and there is nothing relative about it.

If the inner cylinder were at rest whilst the outer one continued to rotate and an observer situated on the outer cylinder were asked whether the inner cylinder was rotating or not, he would reply that the inner cylinder was moving with a velocity v relatively to him but, since every point on it had the same velocity, it was certainly not rotating, although the centre of it appeared to be rotating about him with the same angular velocity as that of the outer ring. If he were a very intelligent observer he might notice that the velocity v corresponded to his own observed angular velocity and conclude that the inner cylinder was not only not rotating but was at rest.

We need hardly explain that this all arose out of the hoary old question of the rotating cylindrical bar magnet. Unnecessary difficulties are introduced by failure to realize the absolute nature of rotation. If one says that the magnet is rotating it is meaningless to ask, relatively to what? It is rotating in the unchanging magnetic field produced by the magnet itself, and electromotive forces are therefore induced in it with consequent movement of electrons until the distribution of charge sets up electric forces which counteract at every point inside the magnet those induced by the rotation. Any translational movement of the magnet can have no electromagnetic effect on the magnet itself.

A stationary charge near the rotating magnet is situated in an unchanging magnetic field but also in the electric field due to the charges on the magnet and it will experience a force due to this electric field. If the magnet be at rest and the charge be rotated around it, the charge will experience a mechanical force due to its rotation in the magnetic field. If both magnet and charge rotate at the same angular velocity the two forces on the charge will be equal and opposite and the charge will therefore experience no force. This does not alter the fact that both magnet and charge are rotating in an unchanging magnetic field.

In the letter to which we referred above, the writer expresses the belief that the phenomena would appear exactly the same to the observer armed with an electric charge whether he were at rest and the magnet rotating or vice-versa. This opinion seems to be based on the assumption that rotation is purely relative. We have seen that the observer can decide definitely which of the two is rotating. If he finds that he is at rest and the magnet rotating, he is surely justified in applying the ordinary laws of electromagnetic induction to the rotating mass of the magnet which will explain the distribution of charges upon it and the electric field thereby set up. If, however, he finds that the magnet is at rest but that he himself is rotating, he will not credit the magnet with anything beyond the production of the magnetic field in which he is rotating; he will find that the charge that he is carrying will experience the same force as before, but due surely to his rotation in the magnetic field.

A satisfactory explanation of the various phenomena is hardly possible unless one realises the absolute nature of rotation. Although one may feel justified in saying that the two cylinders in Fig. 2 are not rotating relatively to one another, it must always be borne in mind that an observer on either cylinder would report that the other one was rotating, and that if one cylinder is at rest, an observer on either can say quite definitely which one is at rest and which rotating. What we said in 1935 about the rotation of the magnet not being relative to anything else, "not even to the observer" would have been more correctly expressed if we had said that the rotation was relative to everything else irrespective of the observer.

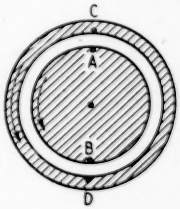


Fig. 2.

G. W. O. H.

MOTOR-CAR IGNITION INTERFERENCE*

The Impulsive Radiated Field

By C. C. Eaglesfield

SUMMARY.—A simple theory is given of the mechanism of motor-car ignition interference. Each spark gives a radiated field of the form of an impulse of easily estimated area and extremely short duration. A car gives a train of such impulses for each nominal spark. The theory seems to fit the known facts.

CONTENTS

1. Introduction.
2. A Simple Theory.
   - Spark Gap as Switch.
   - The Initial Current.
   - The Radiated Field.
   - Duration of Pulse.
3. The Magnitude of the Radiated Field.
   - Receiver Impulse Response.
   - Numerical Values.
   - Comparison with Measurement.
4. The Waveform of the Radiated Field.
5. The Impulse Train.

1. Introduction

MOTOR-CAR ignition can be an important source of interference with radio reception, in particular of television, where it is familiar as spots on the picture and a kind of machine-gun fire from the loudspeaker.

The mechanism by which the interference is caused does not seem to have been very well understood. The usual explanation, as implied for instance by Gill and Whitehead[1] seems to be that the spark causes oscillatory currents in the ignition lead, covering a spectrum of frequencies. This spectrum is then propagated from the ignition lead, considered as an aerial, and the receiver picks out those frequencies which lie within its pass-band.

This picture is hardly satisfactory, since it is known that the spectrum involved is enormously wide, and without gaps. It therefore seems necessary to imagine that the ignition lead has a very large number of resonances, very closely spaced.

A different picture is presented here, very simply derived, which gives a result which seems to fit the facts. The inference is that the radiated field is an impulse, short compared to the period of the carrier frequency up to frequencies of hundreds of Mc/s. Thus what comes from the receiver is the impulse response of the receiving aerial and the receiver.

Since, in general terms, the impulse response is much more connected with the "bandwidth" than with the carrier frequency, it is easy to see how receivers tuned to a wide variety of carrier frequencies are all subject to the interference.

With this simple picture, we need not consider, until we come to the effect on the receiver, things which are characteristic of the receiver and not of the interfering source, namely carrier frequency, type of modulation, etc.

The application of this theory is not limited to motor-car ignition, but applies to all forms of interference caused by sparks. However it is particularly suited to the motor-car, which is a compact, self-contained unit.

The details are given in the next section, ending in a simple formula for the impulsive field, in terms of easily estimated factors. In Sections 3 and 4 the magnitude and waveform of the field as given by the theory are checked against experimental evidence.

Section 5 is devoted to some experiments with cars, the main interest of which is that cars give a train of impulses for each nominal spark.* It seems that each nominal spark is in reality a train of sparks, the number of

* MS. accepted by the Editor, March 1946.

* Professor Howe has kindly sent me a paper—I. "The Efficiency of Generation of High-Frequency Oscillations by means of an Induction Coil and Ordinary Spark-Gap." II. "An Oscillographic Study of some Induction Coil Phenomena," by Professor Howe and J. D. Peattie. *Proceedings of the Physical Society of London*, Vol. 24, Part 5, p. 317, Aug. 15th, 1912, in which there is a description of a similar phenomenon, and a spark photograph (Fig. 24) showing consistently 4 or 5 sparks per break.

sparks in the train and its duration being connected with the oscillatory nature of the ignition systems in use.

The experiments show that the impulse train is a very variable thing; however, they give an indication of the magnitude of the effect, which can greatly modify the annoyance caused by the interference.

2. A Simple Theory

The radiated field from an idealized system will now be calculated. To do this a number of assumptions will be made: the sparking plug will be replaced by a switch; the radiating part of the ignition system will be taken as a small loop having small inductance; and this loop will be supposed to lie in a vertical plane and be very close to the plane of the earth.

The purpose is merely to obtain a first approximation to the radiated field.

The method is to calculate the initial current in the loop by means of the assumption that the plug is a switch; the radiated field is then calculated from the standard formulae for an ideal magnetic dipole.

This treatment leads to the result that the field is an impulse, and the area of the impulse is given by a simple formula.

An impulse is an abstraction: its time of duration is vanishingly small and its amplitude is infinite. In practice the radiated field can only approximate to an impulse, that is, be a pulse of short but finite duration.

It is necessary to keep this point in mind, and have some estimate of the likely duration of the pulse.

Spark Gap as Switch

When the voltage across a gap is gradually increased a critical voltage will be reached at which the gap breaks down. In a very short space of time the gap changes from almost a perfect insulator to almost a perfect conductor.

There is, therefore, no difficulty in replacing it by a switch which is supposed to close when the voltage exceeds a critical value. The only point to be settled is the order of the time interval between the state of non-conduction and the state of conduction.

This interval is certainly very short: the reader is referred to Wheatcroft[2] for a discussion of what happens in the gap during the change.

Fig. 1 is a reproduction of his Fig. 43, which shows a typical curve of the voltage across a gap. The point A is where the change of state occurs; on the left of A the gap is non-conductive, and on the right it is conductive. An abrupt discontinuity of slope at A would indicate zero transition time; actually there is a large localized

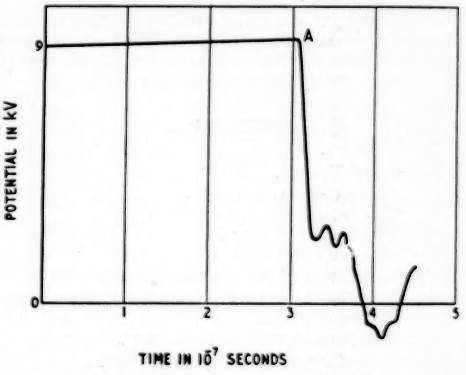


Fig. 1. *Reproduction of Fig. 43 of Wheatcroft, "Gaseous Electrical Conductors," which itself was reproduced from Rogowski and Klemperer, "Arch. f. Elekt.," Vol. 24 (1930).*

curvature, suggesting a very short transition time.

While it is evident from Fig. 1 that the transition time is much less than 10^{-7} sec, it is better to consider the mechanism of the spark for a closer estimate. This does not appear to be fully understood, but the probable course of events seems to be that in the critical condition a small number of electrons leave the cathode and proceed towards the anode, splitting off electrons from the molecules of gas as they go; the released electrons also move towards the anode and split up more molecules; so that an avalanche of electrons descends on the anode.

Until the crest of the avalanche reaches the anode the external current is very small; when the crest arrives the current is very large.

It is known that between the application of a voltage, greater than critical, and conduction a time elapses of the order of the electron transit time across the gap, i.e. about 10^{-8} sec. The application of the voltage presumably corresponds to the electrons leaving the cathode. Thus the transition time between non-conduction and

conduction must surely be small compared to the electron transit time.

It can hardly be greater than 10^{-9} sec.

The Initial Current

Now consider the circuit of Fig. 2 (*a*). The inductance L is supposed to reside in the ignition lead of the car between the sparking plug and say the terminal of the magneto or sparking coil. At that point one can imagine an effective lumped capacitance completing the circuit.

The voltage is supposed to rise comparatively slowly to the critical value V, when the switch closes. At this instant the voltage V is applied across the inductance L and the current, until then zero, rises. The voltage will be considered to remain constant during the initial rise of current.

Immediately after breakdown the current is

$$i = \frac{V}{L}t$$

where t is the time measured from the instant of breakdown. The current is thus as shown

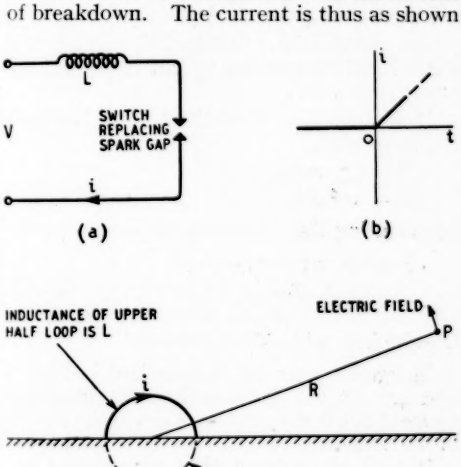


Fig. 2. (*a*) *The equivalent circuit of the spark gap,* (*b*) *the current near breakdown at* $t = 0$, (*c*) *diagram illustrating the propagation.*

in Fig. 2(*b*), and can be expressed as

$$i = \frac{V}{Lp}\mathbf{1} \quad .. \quad .. \quad .. \quad (1)$$

where $\mathbf{1}$ is the Heaviside step operator, and p is the differential operator d/dt.

The Radiated Field

The next step is to consider the propagation between a car and a receiving aerial. We can imagine the ignition lead as a vertical loop, returned to the engine as described, by the plug at one end and the lumped capacitance at the other. If we then consider the engine as part of an infinite conducting plane representing the earth, the situation is as shown in Fig. 2(*c*), where the image of the ignition lead is shown dotted. We now have a closed loop, and for calculation of the field above earth we can consider the whole loop as a magnetic dipole.

Taking the area of the whole loop as A, the magnetic moment is :—

$$m = Ai \quad .. \quad .. \quad .. \quad .. \quad (2)$$

where i is given by equation (1).

The field around a magnetic dipole is given in textbooks on electromagnetic theory, for instance, Stratton[3]; (the formulae as given by Stratton are for a source varying sinusoidally with time).

Referring to Fig. 2 (*c*), the electric field at a point P at a distance R from the loop and in the same plane (R very large) is in the direction shown and is given by :—

$$\frac{\mu}{4\pi cR} \cdot \frac{d^2m}{dt^2} \quad .. \quad .. \quad .. \quad (3)$$

where c is the velocity of propagation ($= 3 \times 10^8$ metres/sec) and μ is the permeability ($= 4\pi \times 10^{-7}$ henry/metre).

Combining equations (1), (2) and (3) we get for the electric field :—

$$\frac{\mu}{4\pi c} \cdot \frac{AV}{LR} p\mathbf{1} \quad .. \quad .. \quad .. \quad (4)$$

Since $p\mathbf{1}$ is a unit impulse, the electric field is an impulse of area :—

$$\frac{\mu}{4\pi c} \cdot \frac{AV}{LR} \quad \frac{\text{volt}}{\text{metre}} \text{sec} \quad .. \quad .. \quad (5)$$

Duration of Pulse

In an actual case the radiated field will be a pulse rather than an impulse. The two factors that seem most likely to lengthen the pulse are lack of instantaneity in the switch action of the sparking plug and the finite length of the ignition lead.

The transition time in the plug has been estimated to be of the order of 10^{-9} sec.

The length of the ignition lead divided by "c" seems an appropriate measure of

the effect of the lead: this would be of the order of 10^{-9} sec.

A reasonable estimate of the duration of the pulse in practice would therefore seem to be of the order of 10^{-9} sec.

3. The Magnitude of the Radiated Field

It is interesting to compare the magnitude of the radiated field as given by equation (5) with published measurements of motor-car interference.

Such results have been given by George.[4] He measured the peak output from a receiver for a number of cars and treated the figures statistically. From a knowledge of the gain of the receiver and aerial he deduced a figure for the peak field strength corresponding to his measured peak output. The receiver was tuned to a number of frequencies in turn, the extreme frequencies being 40 and 450 Mc/s.

To make a comparison with George's figures it is necessary to estimate the impulse response of a receiver. This will be done in such a way that an equivalent field strength is obtained which can be compared directly with George's peak field strength.

Then suitable numerical values must be assigned to the quantities in equation (5).

Some remarks will be added regarding the validity of the comparison.

Receiver Impulse Response

The impulse response of a network can only be rigorously calculated if the network is completely specified, either by its physical components or by its amplitude and phase characteristics.

The impulse response is likely to be a pulse, and if the network has a fairly definite pass-band, it is possible to estimate the peak value of the pulse in terms of the "bandwidth" of the network. This is often justified mathematically by considering a network with arbitrary ideal characteristics, for example see Guillemin.[5] But since the assumed characteristics are mathematically incompatible, it is better to regard the formula obtained as empirical and justified by experience.

There can be little objection to using such a formula here, considering the number of approximations already made.

The method is to multiply the impulse area of equation (5) by the receiver bandwidth, and so obtain an equivalent sinusoidal field strength which gives the same peak output from the receiver.

Doing this, we have:—

$$\text{Equivalent electric field} = \frac{\mu}{4\pi c} \cdot \frac{AVB}{LR} \text{ volts/metre} \quad .. \quad (6)$$

where B is the bandwidth of the receiver.

Equation (6) then gives a figure which can be compared directly with George's figures.

Numerical Values

We now need numerical values for A, V, L.

Suppose that the ignition lead is a semi-circle of diameter 12 inches and wire diameter 0.04 inch; these figures seem reasonable for actual cars.

The area of the whole loop of Fig. 2 (*c*) is then $A = 0.073$ sq. metre.

The inductance of the half-loop can be estimated approximately by taking half the inductance of the whole loop, for which there is a well-known formula (e.g. see Terman[6]).

Then $L = 0.55 \times 10^{-6}$ henry.

The breakdown voltage V, under the conditions in the engine cylinder, is probably about 5000 volts.

And for the conditions of George's measurements.

$R = 33$ metres

$B = 10^4$ c/s

Substituting these figures in equation (6):—

$$\text{Equivalent electric field} = 67 \text{ microvolts/metre} \quad .. \quad (7)$$

Comparison with Measurements

George's results are summarized in Figs. 3 and 4.* It will be seen that the calculated figure of 67 microvolts/metre lies in the middle of his measured values.

It will be noticed that his results are substantially independent of the carrier frequency to which the receiver is tuned. This is in agreement with the theory, which leads to an equation that does not contain the carrier frequency, that is equation (6), and seems to justify the treatment of the radiated field as an impulse even when the receiver has such a high carrier frequency as here, namely 440 Mc/s.

* These are his Figs. 2 and 3 reproduced by permission of the Institute of Radio Engineers.

There is a complication about the polarization. The theory implies vertical polarization of the field whereas George's results show that the field is not vertically polarized. The plane of the ignition lead was taken as vertical because that simplified the calculation of the radiated field. In practice, of course, the plane would probably be different for the several plugs of one car, and certainly different for different cars. It is therefore very unlikely that a definite polarization would be observed when a number of cars were tested.

There are two other points about this comparison. The screening effect of the bonnet, etc., of the car has not been taken into account. Also, as will be shown in the next section, one nominal spark is actually a train of sparks. This would not affect the argument if the receiver bandwidth were great enough to keep the effects of the sparks separate. But it is probable that with the bandwidth of George's measurements the several pulses in the train would be amalgamated into a large pulse greater than the response to a single impulse. These two effects would tend to oppose each other.

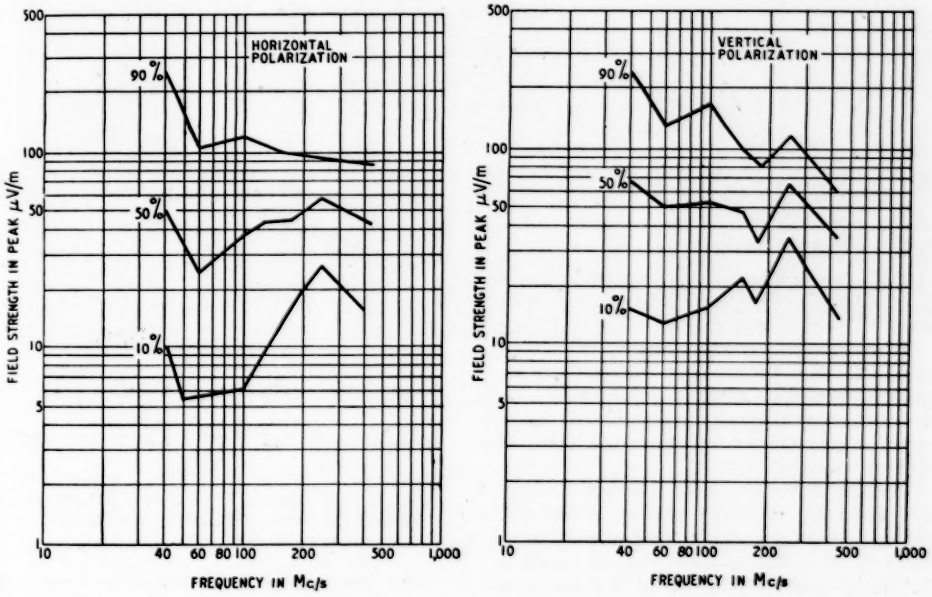
On the whole it may be said that the agreement between the theory and the measurements is satisfactory.

4. The Waveform of the Radiated Field

The comparison of the last section has shown that the theory gives the correct order of magnitude of the radiated field. It remains to verify the waveform.

It is clearly not possible to exhibit on an oscillograph a pulse of the anticipated order of length (10^{-9} sec). The best that can be done is to examine the output from a wide-band receiver: if the output corresponds to the estimated impulse response of the receiver, that is strong confirmation, but not proof, that the input is an impulse. The difficulty is that the receiver impulse response is so much longer than the supposed input pulse.

An experiment was made to test this point by putting a sparking plug near a television receiver, and observing the effect on the



Figs. 3 and 4. *Motor-car ignition radiation, horizontal polarization (left) and vertical polarization (right). Peak field strength versus frequency for a 10-kc/s band. 90, 50 and 10 per cent of all cars and trucks produce less than the field strength indicated by the curves. Receiving aerial 35 feet high and 100 feet from road.*

raster. The receiver was a conventional one, being tuned to 45 Mc/s and having an impulse response of about 0.5×10^{-6} sec.

The plug was connected as in Fig. 5, the capacitance C being charged from a source of high voltage through the resistance R. When the voltage across C reaches the breakdown voltage of the plug, C is discharged and the cycle repeats. The leads connecting C to the plug were very short.

The effect of these sparks was to produce small intense spots on the raster, one spot for each spark.

The size of each spot corresponded to the estimated impulse response of the receiver.

While, as already remarked, this experiment does not prove that the radiated field from the plug is an impulse, it does at any rate show that the effect on the receiver is the same as if it were.

5. The Impulse Train

With the simple arrangement described in the last section it was found that each spark produced one impulse.

Experiments were next made with actual cars. It was found that the interference from a car was more complicated: each nominal spark (each firing of an engine cylinder) gave rise to a train of pulses in the receiver output.

It seems extremely probable that the effect is caused by multiple sparks taking place for each nominal spark, presumably due to oscillations in the sparking coil or magneto; actual ignition systems being of course much more complicated than the simple system of Fig. 5, and having probably several resonant frequencies. The spacing of the plug points may be an important factor, since if the available voltage from the ignition system

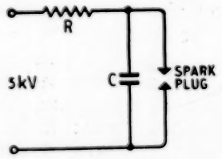


Fig. 5. *A simple circuit for producing sparks.*

does not greatly exceed the breakdown voltage of the plug, multiple sparks are less likely.

A faulty capacitor across the make-and-break contacts might aggravate the effect, but this capacitor was not faulty on the car giving the most pronounced train (car A). The same car was tried with the petrol supply cut off and the engine rotated by the self-starter (itself not a cause of interference): the result was much the same, so the explosion of the petrol does not seem to be involved.

It was found that the length of the train and the number of pulses in it varied from car to car, from plug to plug of the same car, and even on the same plug.

Each individual pulse seemed to approximate to the impulse response of the receiver.

Only one method of obtaining information about the pulse train was found practicable: to isolate a train and record it photographically.

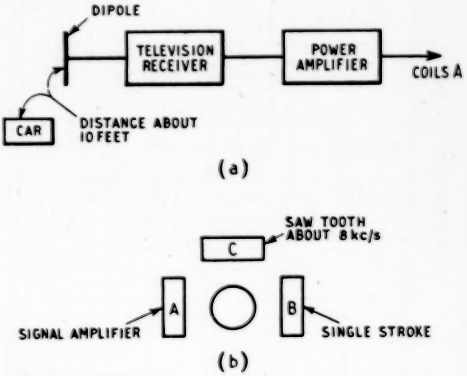


Fig. 6. (*a*) *General arrangement for investigating the pulse train,* (*b*) *the arrangement of the deflecting coils.*

The arrangement of the experiment is shown in Fig. 6 (*a*), the radiation from the car under test being picked up by a television receiver (45 Mc/s), the output of which, when amplified, caused a deflection on a second cathode-ray tube, from which the photographs were taken.

The short distance between the car and the aerial did not modify the results, and helped to reduce the effect of other casual interference.

The arrangement of the deflecting coils on the second cathode-ray tube (the tube in the television receiver was used only as a monitor) is shown in Fig. 6 (b). Coil C was connected to a saw-tooth source (about 8 kc/s); this made the spot oscillate continuously in a horizontal direction. Coil B was used to move the spot from off the screen at the top to off the screen at the bottom at more or less uniform speed. This operation was controlled by a manual switch,

the effect being a single stroke raster, on to which vertical deflection from coil A was superimposed.

The car engine was run at a fairly high speed, about 1000 r.p.m., so that at least one pulse train was caught during each arbitrarily started exposure.

Valuable hints on this type of photography are given by Hendry[7] and Nethercott[8]. The first reference also described the single stroke raster. Since the exposure was timed by the manual switch, the shutter was left open and the camera was placed in a tunnel facing the cathode-ray tube. The camera used was a Rolleicord with double proxar lenses,

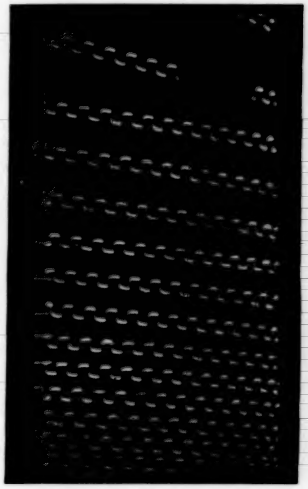


Fig. 7. *Record showing the response of the apparatus to a square wave* (100 *kc/s*).

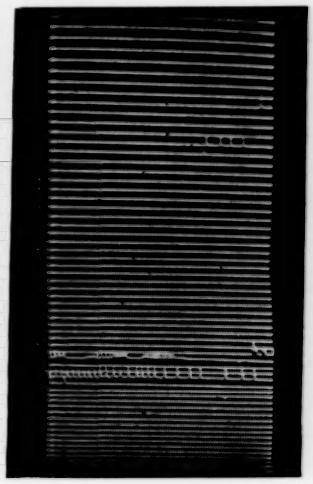


Fig. 8. *A typical record from car A, showing two separate pulse trains.*

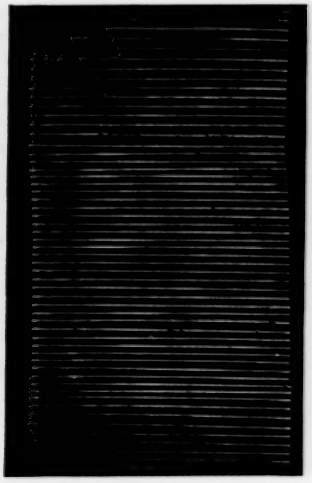


Fig. 9. *A typical record from car C showing two separate pulse trains.*

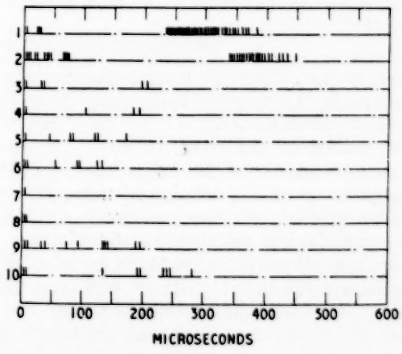


Fig. 10. *A transcription of some of the records, showing the spacing in time of the pulse trains, but not the relative amplitudes of the pulses,* 1, 2 *from car A;* 3, 4 *from car B;* 5, 6 *from car C;* 7, 8 *from car D;* 9, 10 *from car E.*

the tube was a 9-in television tube with a white screen (5 kV on the final anode) and the film was Selo HP3 (Hypersensitive panchromatic).

The response of the system to a square wave (100 kc/s) is shown by Fig. 7; this also serves as a time calibration.

Typical records from cars are shown in Figs. 8 and 9, which each contain two separate pulse trains.

As suggested by Hendry[7] it was found better to use a not so bright but well focussed spot on the cathode-ray tube and intensify the negative after development. This can be seen by comparing Fig. 9 which was intensified with Figs. 7 and 8 which were not.

Due to overloading in the amplifier, the amplitudes of the pulses in the records are not reliable. The difficulty was to put a large enough current into the deflection coil, which had to be made of few turns to retain definition.

Typical records from five cars have been transposed to a linear time scale in Fig. 10. This shows the spacing in time of the pulse trains, no account being taken of amplitude.

The following table of results gives an idea of the kind of pulse train that can be expected.

Car	No. of Trains recorded	Average No. of pulses per train	Average duration of train
A. Morris 8 ..	6	50	500 sec.
B. Hillman 10..	5	3	100 ,,
C. Vauxhall 10	8	10	125 ,,
D. Hillman 10 ..	4	5	50 ,,
E. Ford 10 ..	9	10	200 ,,
Average ..		15	200 ,,

Conclusions

A simple theory of the impulsive radiated field of motor-car ignition has been given by considering the sparking plug as a switch and then calculating the current and the radiated field and estimating the likely duration of the pulse ; the magnitude of the radiated field has been compared with published measurements by estimating the receiver impulse response and inserting likely numerical values in the theoretical formula, and the waveform checked by experiment. Experiments with cars have shown that they give an impulse train.

REFERENCES

[1] Gill and Whitehead. "Electrical Interference with Radio Reception." *Journ. Instn Elec. Eng.*, Vol. 83 (1938).

[2] Wheatcroft. "Gaseous Electrical Conductors" (especially Chapter VIII).

[3] Stratton. "Electro-Magnetic Theory" (Section 8-6).

[4] George. "Field Strength of Motor-car Ignition between 40 and 450 Megacycles." Sept. 1940. *Proc. Inst. Rad. Engr.*

[5] Guillemin. "Communication Networks" (Chapter XI).

[6] Terman. "Radio Engineer's Handbook" (Section 2.9).

[7] Hendry. "Photography of Cathode Ray Tube Traces." *Electronic Engineering*, Jan. 1944.

[8] Nethercott. "The Recording of High Speed Transient Phenomena." *Electronic Engineering*, Feb. 1944.

DESIGN OF ATTENUATION EQUALIZERS*

Generalized Performance Equations

By H. N. Wroe, B.Sc., Ph.D.

(Assistant Engineer, Department of Post and Telegraphs, Eire)

SUMMARY.—From theoretical considerations a method of accurately calculating the performance of a given equalizer is deduced. This is utilized and exemplified in the design of equalizers which, because they are not constructed on conventional lines, do not lend themselves to customary methods of design calculation.

Introduction

IN reaction, as it were, from the complexity of earlier expositions of the principles governing equalization in which the subject was presented in a generalized form rather unsuitable for practical application, later treatments have tended to concentrate on particular types of equalizer having special internal arrangements—usually the constant impedance (resistance) form—which for the very reason that they simplify mathematical considerations, mask the general factors on which equalization depends. A further consequence is that information on the prediction of the performance and on the design of very simple equalizers is lacking because they do not possess the internal arrangement normally assumed. In spite of their simplicity, therefore, such equalizers are usually not at all the easiest to deal with.

Such considerations may not be of great moment to the specialist in network design who, backed by the resources of a great manufacturing corporation which will provide almost any components he may desire, is occupied almost entirely with the attainment of first-class transmission quality. To the ordinary communications engineer, however, the following of conventional designs frequently becomes difficult, if not impossible. Despite frequent assumptions to the contrary, he is often called upon to provide transmission networks in circumstances where considerations of cost or of supply are the

* MS. accepted by the Editor, February 1946.

determining factors, and where the use of the minimum of standard components or, perhaps, of whatever is to hand, is necessary. For instance, he may design a constant-impedance equalizer only to find that the components available have, either nominally or owing to manufacturing tolerances, values appreciably different from those which the design demands. A method of predicting whether, despite this, the resulting quality of transmission will be of sufficiently high standard for the required purpose, or whether it can be made so by alterations, would be very useful. Or again, a small amount of equalization which could readily be provided by a very elementary form of equalizer, may be required; the difficulty is to work out a basis for design, or if recourse be had to trial and error, to calculate the performance of any given assumed network.

It is to deal with the deficiencies outlined above, and to meet in a general way the needs of the non-specialist, that this paper has been written. It is felt that an exposition of the general principles determining equalizer performance illustrated in their application to telephone cables will be of some value.

As the limits for good quality transmission over the important portion of the frequency range are narrow, the methods of calculation must avoid all but really negligible approximations. This involves additional labour in computation, but if it enables calculation at the desk to replace trial and error adjustments in the field, or even if it only provides a guide to these, it will be well worth while.

General Considerations

As is well known, it is characteristic of practical transmission lines that their transmission losses vary with frequency, the losses generally increasing with frequency. In telecommunication circuits, where the signals extend over a band of frequencies, this means that the relative magnitudes of signals (and of signal components) having different frequencies are altered during the passage of the signals along the line, the result being a mutilation known as frequency distortion. For faithful reproduction of the original signals at the receiving terminal it is necessary to correct for this distortion by the process known as equalization. This consists of connecting in the circuit a correcting network, or equalizer, the function of which is to make the resultant overall loss invariable with frequency over the frequency band required.

In order that this object be attained the loss-frequency characteristic of the equalizer must be inverse to that of the line; that is, the transmission loss of the equalizer must fall with frequency. Clearly, then, the basic circuit of the equalizing network will take the form of a four-terminal network having as series arm a capacitor, and as shunt arm an inductor. In designing the equalizer the primary aim is to arrange that its transmission losses at all points within the frequency range to be covered are such that when combined with the corresponding line losses the total loss is independent of frequency.

The general practice now is to connect the equalizer at the receiving terminal of the line; i.e., to employ post-equalization.

Performance Equation

The theoretical treatment given below is based on the following premises:

1. Only frequency distortion is to be eliminated.

2. Only distortion produced by the communication circuit itself is to be considered—the transmitter and receiver, or their equivalent in the form of repeaters (valve amplifiers), being regarded as ideal.

3. Insertion loss measurements are a reasonable criterion of transmission quality.

4. Post-equalization only is employed.

5. Transmission through the equalizer is unidirectional. This assumption is made mainly for convenience.

The elements of a generalized unidirectional communication circuit are shown in

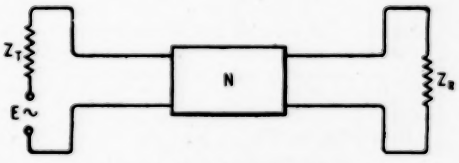


Fig. 1. *Generalized communication circuit N connected between impedances Z_T and Z_R.*

Fig. 1, where the circuit is represented by an equivalent network N, working between a transmitter of impedance Z_T, and a receiver of impedance Z_R. Usually in transmission measurements $Z_T = Z_R = R \angle 0$, the latter

normally being 600 $\angle$ 0 ohms. Repeaters are generally constructed to act as transmitters and receivers operating under this condition.

If the curve AB of Fig. 2 represents the insertion loss characteristic of network N alone (measured for terminating impedances Z_T, Z_R) the circuit loss curve CD may, roughly speaking, be taken as the ideal goal of equalization, the point D being fixed by f_c, the maximum frequency in the range over which equalization is desired. On this basis, at any frequency within this range the equalizer must introduce into the circuit a loss x which, added to the network loss w, gives the quantity y. In practice the resultant characteristic will have the form of the curve EF, the overall insertion loss of the equalized circuit at any frequency f, being denoted by A_0. What is needed is an equation enabling the value of A_0 to be calculated at any frequency. This will be required, firstly, in determining the magnitude of the equalizer components (when A_0 will be assumed to coincide with y), and secondly in predicting the overall circuit quality these will produce.

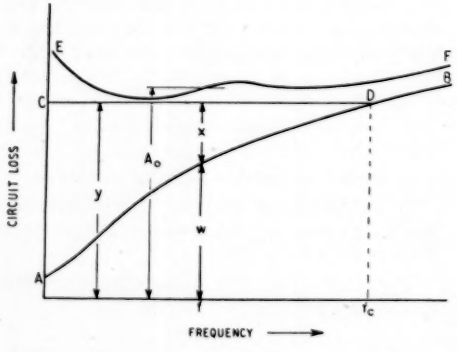


Fig. 2. *The network loss is represented by AB and the desired performance by CD; a practical resultant is more in the nature of EF.*

Referring to Fig. 3 let

E = sinusoidal transmitter e.m.f.

Z_I', Z_I'' = image impedances of network N.

θ = transfer constant of network $N = \alpha + j\beta$.

Z_E = impedance presented to N by the equalizer terminated in Z_R.

I_1 = current to Z_R when directly connected to Z_T without any intervening circuit.

I_2 = current to Z_R.

I_3 = current to equalizer.

$K = \dfrac{I_3}{I_2}$ where K is, in general, complex.

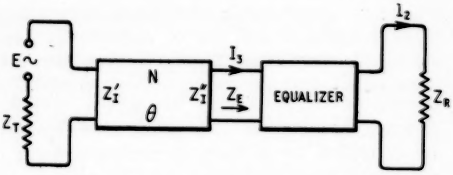


Fig. 3. *Generalized communication circuit including an equalizer.*

Any network worth equalizing will almost invariably (at any rate for the important frequencies) be electrically "long", so that interaction loss may be neglected.* Regarding Z_E for the moment as a receiver, the following relationship, which may be found in several textbooks on transmission, then holds:

$$I_3 = \frac{E}{Z_T + Z_E} \times \frac{Z_T + Z_E}{\sqrt{4Z_T Z_E}} \times \frac{\sqrt{4Z_T Z_I'}}{Z_T + Z_I'} \times \frac{\sqrt{4Z_E Z_I''}}{Z_E + Z_I''} \times e^{-\theta} \quad .. \quad .. \quad (1)$$

We have also

$$I_3 = KI_2 \quad .. \quad .. \quad .. \quad .. \quad (2)$$

$$I_1 = \frac{E}{Z_T + Z_R} \quad .. \quad .. \quad .. \quad (3)$$

Substituting (2) and (3) in (1) we obtain the insertion loss of the network N plus the equalizer (i.e., A_0) as

$$A_0\,(\text{db}) = 20 \log_{10} \frac{I_1}{I_2}$$

$$= 20 \log_{10} \left| \frac{Z_T + Z_E}{Z_T + Z_R} \right| - 20 \log_{10} \left| \frac{Z_T + Z_E}{\sqrt{4Z_T Z_E}} \right| + 20 \log_{10} \left| \frac{Z_T + Z_I'}{\sqrt{4Z_T Z_I'}} \right| + 20 \log_{10} \left| \frac{Z_E + Z_I''}{\sqrt{4Z_E Z_I''}} \right| + 20 \log_{10} |e^{\theta}| + 20 \log_{10} |K|$$

$$.. \quad .. \quad (4)$$

From this equation the overall performance of the equalizer represented by A_0 may

* See Appendix.

be calculated. It is seen to depend not only upon the current-changing property of the equalizer (represented by K), which may be termed the " potentiometer " effect, and on that denoted by θ, but also on several other components of a more or less complex nature—reflection losses at the network terminals, and losses produced by inequalities between Z_T, Z_R, and Z_E.

Uniform Lines

With uniform transmission lines such as telephone cable circuits,† we have

$$Z_I' = Z_I'' = Z_0,$$

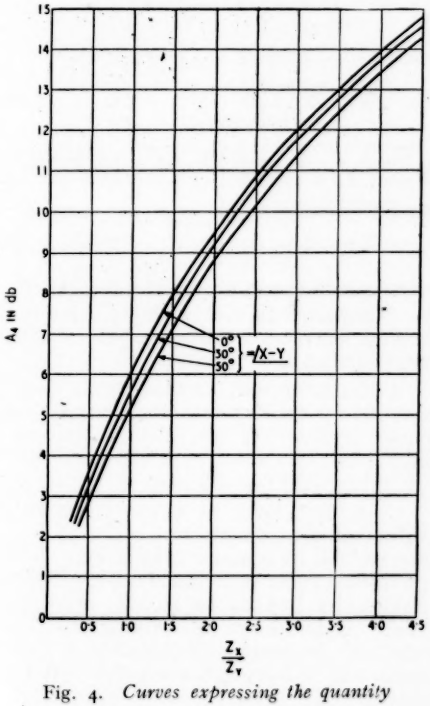


Fig. 4. *Curves expressing the quantity* $20 \log_{10}\left|1 + \frac{Z_x/x}{Z_y/y}\right|$

where the latter is the line characteristic impedance, and θ, α, and β, now become the well-known propagation, attenuation, and phase constants respectively (for the whole line, not per unit length). If in addition, as is almost invariably the case, $Z_T = Z_R = R/\underline{0}$ (4) becomes

$$A_0 = \underbrace{20 \log_{10}|e^{\alpha}|}_{A_1} + \underbrace{20 \log_{10}|K|}_{A_2} + \underbrace{20 \log_{10}\left|\frac{(R+Z_0)^2}{4RZ_0}\right|}_{A_3} + \underbrace{20 \log_{10}\left|\frac{Z_0+Z_E}{Z_0+K}\right|}_{A_4} \quad .. \quad .. \quad (5)$$

Now $A_1 + A_3 =$ the insertion loss of the line for terminating impedances $R/\underline{0}$, and is equal to w (Fig. 2), A_3 being the sum of the terminal reflection losses.

Hence

$$A_0 = w + A_2 + A_4 = w + 20 \log_{10}|K| + 20 \log_{10}\left|\frac{Z_0+Z_E}{Z_0+R}\right| \quad .. \quad .. \quad (6)$$

Thus for a given line and given terminating impedances the equalizer performance depends upon two quantities, K and Z_E, and can be calculated by means of (6) without making conventional assumptions as to the internal structure of the equalizer. Much labour in computation can be saved by treating A_4 as follows :—

Write $\frac{Z_0+Z_E}{Z_0+R} = \frac{1+Z_E/Z_0}{1+K/Z_0}$

Then

$$A_4 = 20 \log_{10}|1+Z_E/Z_0| - 20 \log_{10}|1+R/Z_0|$$

These two quantities can be read off from the curves of Fig. 4, in which the quantity $20 \log_{10}\left|1 + \frac{Z_x/x}{Z_T/y}\right|$ is plotted against Z_x/Z_T for different values of $/x - y$.

Design

The general method may be illustrated by taking the simple ladder type of equalizer sketched in Fig. 5. The treatment falls naturally under two headings :

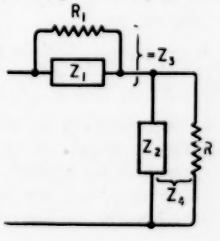


Fig. 5. *Simple ladder-type equalizer.*

† The theoretical treatment given here applies, of course, to uniform lines in general; but cable circuits are particularly considered because practical data for them are available in this paper.

1. *Constant-Impedance Equalizers*

These have been the subject of considerable notice during the past few years, and consequently will be considered here in bare outline only. The characteristic property is $Z_E = R/o$ at all frequencies, and inserting this in (6) it will be found that A_4 vanishes, and we obtain

$$A_0 = w + 20 \log_{10}|K| \quad .. \quad .. \quad (8)$$

Here we want $A_0 = y$ (Fig. 1) and so

$$20 \log_{10}|K| = y - w = x,$$

or $$|K| = \text{antilog}_{10}\, x/20 \quad .. \quad .. \quad (9)$$

K having been determined, Z_2 can be found from the relation

$$\frac{R + Z_2}{Z_2} = K \quad .. \quad .. \quad .. \quad (10)$$

Z_1 then follows from the constant-impedance condition

$$\left.\begin{aligned} Z_1 Z_2 &= R^2 \\ R_1 &= R \end{aligned}\right\} \quad .. \quad .. \quad (11)$$

It will be seen that the constant-impedance relation produces not inconsiderable simplification in the mathematics which is one—though not the only one—of the advantages of equalizers of this type.

2. *Inconstant-Impedance Equalizers*

Here A_4 remains in (6) and we are presented with two unknowns, K and Z_E. A separation can, however, be achieved as follows :—

A_3 is independent of the length of the line, is relatively small, and depends upon relationships between impedances. A_1 varies with the length of the line and is by far the more important. Let the latter, then, be equalized by A_2 alone, and the former by A_4 which resembles it in composition. Proceeding on this basis we may, for the purposes of convenience and illustration, suppose the curve AB (Fig. 2) to be a graph of A_1, and putting x', y', w', for x, y, w, on our new curve, we have

$$20 \log_{10}|K| = x' \quad .. \quad .. \quad .. \quad (12)$$

From this K and Z_2 follow as before.

It remains to determine Z_E. We might plot a curve of A_3 against frequency and proceed as before, putting A_4 equal to the loss required for equalization at any given frequency, but another and more convenient plan which has proved itself in practice is open to us.

The reflection loss term A_3 would be equalized when

$$\frac{(Z_0 + R)^2}{4RZ_0} \times \frac{Z_E + Z_0}{R + Z_0} = \text{a constant.}$$

Now this constant could have any one of an infinite number of values ; but a convenient value would be unity, for then there would be no resultant loss.

Hence writing

$$\frac{(Z_0 + R)^2}{4KZ_0} \times \frac{Z_E + Z_0}{R + Z_0} = 1$$

we obtain

$$Z_E = \frac{Z_0 (3R - Z_0)}{K + 1_0} \quad .. \quad .. \quad (13)$$

Since Z_2 is known, Z_4 (combination of R and Z_2 is parallel) can be calculated, and Z_3 then follows from

$$Z_3 = Z_E - Z_4 \quad .. \quad .. \quad .. \quad (14)$$

A knowledge of Z_3 enables R_1 and Z_1 to be determined as will be shown later in discussing particular configurations of ladder-type equalizers.

It might appear, at first sight, that the method for dealing with the second class of equalizer involves an unusual amount of laborious calculation, and consequently the following observations may be pertinent :—

1. In any form of network design the values of components can scarcely ever be determined by a single calculation. A compromise between optimum values for different frequencies has to be struck, and several trials made before a working solution is reached. Subsequently " trimming " operations will follow, and altogether a not inconsiderable amount of computation is required.

2. Preliminary calculations of values of components need be made at two or three frequencies only, a check of performance over the whole frequency range being necessary only when these have been determined.

3. The effect of variations in the impedances composing A_4 (Eqn. 5) is relatively small, so that Z_0 and Z_E are not required with great accuracy. As standard values of Z_0 for the type of cable used are sufficient,

impedance measurements on the line actually being equalized are unnecessary.

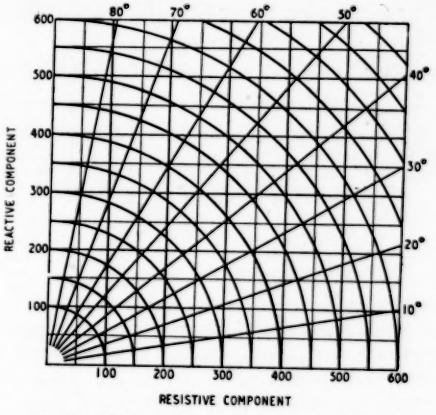


Fig. 6. *Diagram for transposing polar and Cartesian coordinates.*

Similar considerations apply to reflection losses, and it follows that once the frequency — insertion loss characteristic of the line to be equalized has been obtained the calculation of A_1 (Eqn. 5) is a simple matter.

Since Z_E need not be obtained with great precision, the calculations leading up to its determination may be much shortened by the use of a graph of the form shown in Fig. 6, which enables the transformation of impedances from polar to rectangular co-ordinates, and vice versa, to be made very readily.

The calculation of A_2 can be simplified by the use of Fig. 7 where A_2 is plotted against $|Z_2/R|$. Reference has already been made to the use of curves for determining A_4.

4. It will usually be found that several of the quantities calculated at one stage of the design reappear at another. This factor makes for further labour saving, a point which is borne out in the mathematical treatment of two forms of equalizer which are considered below as examples. For reasons already given constant-impedance conditions are ignored.

Practically speaking the simpler the equalizer the cheaper it becomes. Normally, the components must include at least one inductor, and this is by far the most expensive item. The very simple equalizer will, therefore, contain but one of the latter, and the types now to be discussed and sketched in Figs. 8 and 9 are constructed on this basis.

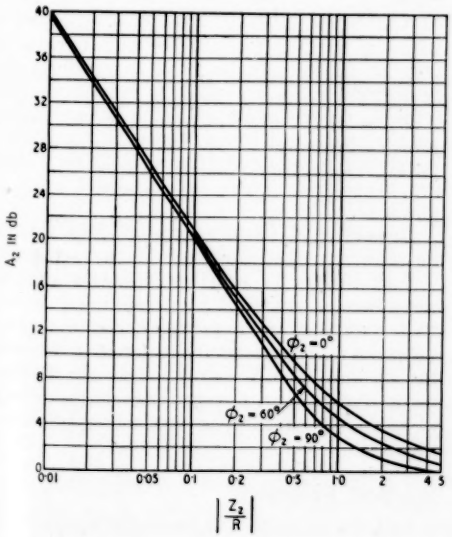


Fig. 7. *The value of A_2 in terms of $|Z_2/R|$; $Z_2 = |Z_2|\underline{\phi_2}$.*

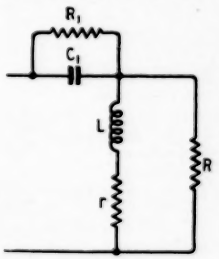


Fig. 8. *Type I equalizer.*

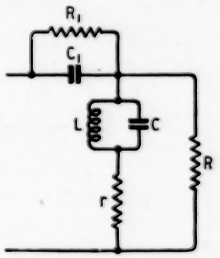


Fig. 9. *Type II equalizer.*

Type I

Here A_2 (Eqn. 5) will not be zero at f_c, so that a quantity, h (= say, 2 or 3 db) must be added to x'.

We then have

$$10 \log_{10}|K|^2 = x' + h$$

or

$$|K|^2 = \text{antilog}\frac{x' + h}{10} = \text{M, say} \quad .. \quad .. \quad (15)$$

Again

$$K = \frac{R + Z_2}{Z_2} = 1 + \frac{R}{Z_2} = 1 + \frac{R}{r + j\omega L} \quad [\omega = 2\pi f]$$

Hence

$$|K|^2 = \frac{(R+r)^2 + \omega^2 L^2}{r^2 + \omega^2 L^2} = M \quad .. \quad (16)$$

In this equation there are two unknowns. To effect a separation, take a very low frequency, i.e., put $\omega = 0$, and let M_0 be the corresponding valve of M.

Then $$\frac{(R+r)^2}{r^2} = M_0$$

and $$r = \frac{R}{\sqrt{M_0 - 1}} \quad .. \quad .. \quad .. \quad (17)$$

Hence r is obtained. It includes any resistance possessed by L, though this is usually negligible in modern inductors for these networks.

L can now be calculated from (16) which gives us

$$L = \frac{\sqrt{(R+r)^2 - Mr^2}}{\omega\sqrt{M-1}} \quad .. \quad .. \quad (18)$$

Z_E is found from (13).

$$Z_4 = \frac{RZ_2}{R+Z_2} = \frac{R(r+j\omega L)}{R+r+j\omega L}$$

$$= \frac{R\sqrt{r^2+\omega^2L^2}\ \underline{/\delta}}{\sqrt{(R+r)^2+\omega^2L^2}\ \underline{/\gamma}} \qquad \delta = \tan^{-1}\frac{\omega L}{r} \quad \gamma = \tan^{-1}\frac{\omega L}{R+r}$$

$$= \frac{R}{|K|}\underline{/\delta - \gamma} \quad .. \quad .. \quad .. \quad (19)$$

Z_3 then follows from (14)

Now $$Z_3 = \frac{R_1 \times 1/j\omega C_1}{R_1 + 1/j\omega C_1} = \frac{R_1}{1 + j\omega C_1 R_1}$$

$$= \frac{R_1}{1+\omega^2 C_1^2 R_1^2}\ \underline{\sqrt{\nu}} \qquad \nu = \tan^{-1}\omega C_1 R_1$$

$$= R_1 \cos\nu\ \underline{\sqrt{\nu}} \quad .. \quad .. \quad .. \quad (20)$$

From this R_1, ν, and hence C_1 can be obtained.

If at the commencement it is clear that r will be small compared with ωL, it may be neglected in determining L which may then be found by means of Fig. 7, the curve for $\phi = 90°$ being employed, and L being calculated from the value of Z_2 ($=\omega L$ here) read off for the required value of A_2.

Type II

This is more suitable for equalizing long lines than is Type I. Moreover, with the latter A_2 can never be zero and, generally speaking, its value at f_c represents an inefficiency in the equalizer which is important in cases where the loss of the equalized circuit approaches the maximum gain of the repeaters employed. In Type II equalizers this is avoided by making L and C resonate at f_c. We then have

$$\omega_c L = 1/\omega_c C \qquad \text{where } \omega_c = 2\pi f_c$$

$$\therefore \quad C = 1/\omega_c^2 L \quad .. \quad .. \quad .. \quad (21)$$

The impedance Z_5 of L and C in parallel is

$$\frac{j\omega L \times 1/j\omega C}{j\omega L + 1/j\omega C} = \frac{j\omega L}{1-\omega^2 LC} = \frac{j\omega L}{1-\omega^2/\omega_c^2}$$

$$= j\omega L/\eta \qquad \text{where } \eta = 1 - \omega^2/\omega_c^2$$

$$= j\omega L' \quad .. \quad .. \quad .. \quad .. \quad (22)$$

where $L' = L/\eta$

Thus the LC combination can, for frequencies up to f_c, be regarded as an inductance L' which varies with frequency. From this point the mathematical treatment is as for Type I, L' taking the place of L. The resistance of the inductor is taken as negligible.

It will be noticed that here a complete break with the constant-impedance type of equalizer has been made, for in this instance the latter demands a second inductor in the series arm.

Trimming

The components lending themselves most readily to trimming alterations are, of course, r and R_1. At low frequencies the influence of the latter predominates over that of L while at higher frequencies the reverse is true so that r can be used to readjust the performance characteristic at low frequencies without appreciably affecting it at higher values, particularly in Type II equalizers. It should be noticed that increasing r does not necessarily make for a reduction in A_2 (Eqn. 5).

Increasing R_1 frequently produces a general flattening of the equalization characteristic.

With Type II equalizers better quality can often be obtained by arranging L and C to resonate at a frequency considerably in excess of f_c. This naturally tends to increase the overall circuit loss, though not to any great extent, and the problem becomes one of balancing this increase against the improvement in quality.

Practical Examples

In Figs. 10 to 12 will be found a number of measured quality curves for experimental audio circuits incorporating equalizers (shown

inset in the Figs.) designed on the lines described above. In none is the constant

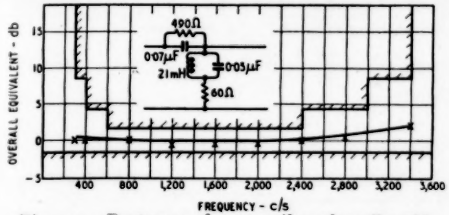


Fig. 10. *Response of 12.7 miles of 20-lb cable with the equalizer shown;*
——— *measured,*
x——x——x *calculated.*

impedance condition employed. R was taken as $600/0$ ohms so that the overall circuit insertion loss is here the overall equivalent. The latter has been plotted with its value at 800 c/s taken as zero, and the points indicate calculated values. In working out the designs the object was to obtain good transmission quality without excessive losses in the equalizers themselves, and with an eye to keeping the overall equivalent within normal repeater gains.

It is not to be supposed that the results are necessarily the best that could have been achieved. They are given mainly to demonstrate that the predicted quality is close to the measured values, and also to indicate the sort of quality which may be expected from very simple equalizers. The size of cable is

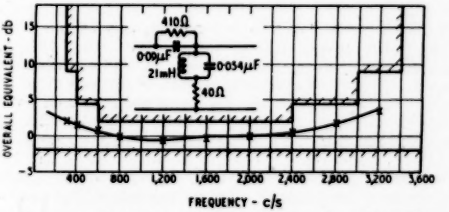


Fig. 11. *Response of 8.85 miles of 10-lb cable equalized;*
——— *measured,*
x——x——x *calculated.*

given in terms of the weight of one mile of single conductor.

Ideal equalization being impossible of attainment some practical criterion is necessary, and the quality limits of the C.C.I.F. for international 4-wire circuits suggest themselves as a convenient widely recognized standard. They are, however, not now representative of the very best practice, and in Figs. 10 to 12 the proposed new values of these limits are indicated by the shaded lines.

Fig. 14 shows the overall equivalent for an experimental equalized music circuit, the shaded lines indicating the C.C.I.F. limits for international music circuits. The circuit of the equalizer employed is shown inset.

Conclusion

It has been shown that the foregoing theoretical treatment enables an accurate prediction of equalizer performance to be made without the assumption of particular forms of equalizer being necessary, and from it a method of designing simple types of equalizer which have no special internal arrangement has been deduced. Equations

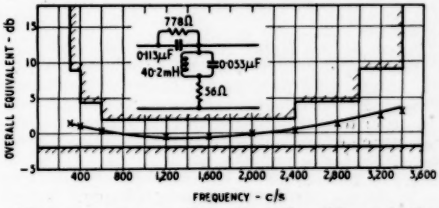


Fig. 12. *19.92 miles of 40-lb cable and the equalizer;*
——— *measured,*
x——x——x *calculated.*

4 and 5 are quite general and enable the estimation of the effect of such extreme forms of equalizing network as a single capacitor or inductor connected in series or in shunt at the circuit terminal to be made, and the not uncommon simple " rejector " type of equalizer to be handled in calculations of performance. There does not appear to be any reason why they should not be applicable to other than audio and music circuits.

Equation (29)—see Appendix—is of particular interest for electrically short lines and in cases where discontinuities in the uniform distribution of electrical line constants arise.

The Author's thanks are due to the Engineer-in-Chief, Department of Posts and Telegraphs, Eire, for permission to publish this paper. He desires also to express his acknowledgements to Mr. B .O'Mongain, A.R.C.Sc.I., B.Sc., for pointing out the deduction of equation (29).

APPENDIX

Importance of Interaction Loss

Since interaction loss is a complex quantity depending on a diversity of factors, a rigorous demonstration that it is, in general, negligible for circuits requiring equalization is not possible. What can be done is to indicate general tendencies,

to appeal to experience, and to cite practical examples.

Thus, if for uniform lines interaction loss is to be included, equation (5) contains an additional term, viz. :—

$$A_5 = 20 \log_{10} \left| 1 - \left(\frac{Z_0 - R}{Z_0 + R}\right)\left(\frac{Z_0 - Z_E}{Z_0 + Z_E}\right) e^{-2\theta} \right|$$
$$= 20 \log_{10} |1 - W| \quad .. \quad .. \quad .. \quad (23)$$

where

$$W = \left(\frac{Z_0 - R}{Z_0 + R}\right)\left(\frac{Z_0 - Z_E}{Z_0 + Z_E}\right) e^{-2\theta}$$

Now A_5 will be negligible if W is small, and this means, in the first place, that the differences $(Z_0 - R)$, $(Z_0 - Z_E)$ must not be excessive. For several transmission reasons Z_0 and R are of the same order of magnitude, while it will be found that, in general, Z_E does not differ (or can be made not to differ) greatly from Z_0. Thus the product

$$\left(\frac{Z_0 - R}{Z_0 + R}\right)\left(\frac{Z_0 - Z_E}{Z_0 + Z_E}\right)$$

is not large.

The remaining factor is $e^{-2\theta}$ and this quantity decreases rapidly with θ; i.e., with the length of the line. In practice, for any but electrically short lines $e^{-2\theta}$ is such as to reduce W to negligible proportions.

In order to give actual figures the interaction loss of an extreme case has been calculated, and is shown plotted against frequency in Fig. 13. It is for 3 miles of 20-lb cable, which for audio circuits is on the borderline for worth-while equalizing. Values of Z_E have had to be assumed ; and since the phase angle of this quantity is usually small for the equalizers discussed above, it has been taken to be zero. R has been assumed to be 600 / 0 ohms.

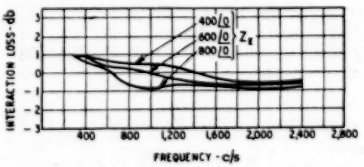


Fig. 13. *Interaction loss for several values of Z_E.*

It will be seen that even in this instance, allowance being made for variations in Z_E with frequency —normally not large—and for the fact that it is relative loss which is of importance, the effect of interaction loss on equalization is not great ; and that, if anything, it is of assistance. Moreover with short lines the total amount of equalization required is small so that adjustments to allow for interaction loss can readily be arranged. Thus even with these lines it is reasonable to design on the basis of neglecting the latter, and then, if necessary, to check back and make any alterations which calculations of interaction loss (a rather laborious process) may indicate as being desirable.

There is, however, an alternative line of attack. Referring to Fig. 3 and to the definition of terms preceding (2), let E_0 be the open-circuit voltage at the receiving end of the network N when the equalizer is disconnected, and let Z_B be the impedance looking back into the network at this point.

Then by Thévenin's Theorem

$$E_0 = I_3(Z_E + Z_B) \quad .. \quad .. \quad .. \quad (24)$$

If the network be now terminated in Z_R, the current, I_4, to the latter is

$$I_4 = \frac{E_0}{Z_R + Z_B} \quad .. \quad .. \quad .. \quad .. \quad (25)$$

Combining (24) and (25) we have

$$I_3 = I_4 \cdot \frac{Z_R + Z_B}{Z_E + Z_B} \quad .. \quad .. \quad .. \quad (26)$$

whence

$$I_2 = \frac{I_4}{K} \cdot \frac{Z_R + Z_B}{Z_E + Z_B} \quad .. \quad .. \quad .. \quad (27)$$

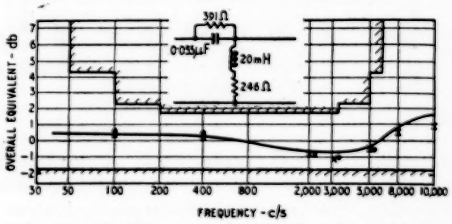


Fig. 14. *Performance of 1.88 miles of equalized 10-lb cable ;*
measured,
calculated A,
calculated B.

The overall insertion loss of the equalized circuit is given by

$$20 \log_{10} \left|\frac{I_1}{I_2}\right| = 20 \log_{10} \left|\frac{I_1}{I_4} \cdot K \cdot \frac{Z_E + Z_B}{Z_R + Z_B}\right| \quad (28)$$

Now $20 \log_{10} \left|\frac{I_1}{I_4}\right|$ is the insertion loss of the network N for impedances Z_T, Z_R, and is therefore w (Fig. 2).

Hence

$$20 \log_{10} \left|\frac{I_1}{I_2}\right| = w + 20 \log_{10} |K| + 20 \log_{10} \left|\frac{Z_E + Z_B}{Z_R + Z_B}\right|$$
$$.. \quad .. \quad (29)$$

This equation is similar to (6), Z_B replacing Z_0 in the latter ; but does not require any assumption as to the magnitude of interaction loss. In (6), however, Z_0 is a quantity which, as we have already seen, can be regarded as standard for the type of line employed, whereas Z_B must be measured over the frequency range required for each particular case ; but for very short lines where interaction loss is considered to be of importance the additional labour may be worth while, and the overall performance can then readily be calculated by means of (29).

For the circuit of Fig. 14 the design was based on (29) owing to the electrical " shortness " of the line. The predicted performance as given by the latter are indicated by the points A.

As a matter of comparison the performance was also calculated by (6), the results being given by points B. It is evident that here the effect of interaction loss is inappreciable.

It is interesting to note that unlike (6) (29) does not depend upon the assumption of uniformly distributed line constants, so that it remains valid for circuits possessing discontinuities such as those produced by the insertion at intermediate points of filters and the like, or by the utilization of a combination of cable and open-wire line, etc., and is therefore of special importance in these cases.

LINEAR FREQUENCY DISCRIMINATOR*

By J. R. Tillman, Ph.D., A.R.C.S.

Introduction

IN a transmission system using frequency modulation (f.m.), the frequency of a carrier wave is modulated by the primary signal, e.g. speech, in such a manner that (a) the rate of change of frequency of the transmitted signal corresponds to the frequency of the primary signal and (b) the instantaneous carrier frequency differs from its unmodulated value (the centre frequency) by an amount proportional to the instantaneous amplitude of the primary signal. The reverse effect, the conversion of frequency- to amplitude-modulation, in the receiver of such a system, is usually carried out by means of a frequency discriminator. Among the important requirements of this section of the receiver are a conversion ratio which is high over the important range of frequencies, and low outside it, and a high degree of linearity of conversion.

The simplest frequency discriminators do not meet one or more of these requirements and are rarely used. Of the more satisfactory designs an early one has two parallel-resonant circuits, A and B, with resonant frequencies f_A and f_B so chosen that $\sqrt{f_A.f_B}$ is the centre frequency of the f.m. signals, and with equal impedances at resonance. The signals are fed to both these circuits and each is connected separately to a diode detector and d.c. load resistor. The resistors are so connected that the rectified voltages are in series opposition. The relationships between frequency and output voltage for each resonant circuit, and between frequency and net rectified output voltage, will then be as shown in Fig. 1, provided a certain calculable relationship between the magnifications (Q) of the tuned circuits and $(f_B-f_A)/\sqrt{f_A.f_B}$ has been satisfied. The central portion of the lower curve is substantially linear and when f_A and f_B are correctly chosen this linear range just embraces all the frequencies which comprise the f.m. signal.

Later designs of discriminator[1] have usually aimed at satisfying more effectively one or other of the requirements already stated. All such designs known to the author have made use of capacitance, resistance and inductance.

Because of the inclusion of inductors, the instability of the characteristics of these discriminators with changes of temperature and ageing is excessive for some purposes unless great care is taken with the design and manufacture of these particular components. The temperature coefficient of inductance of the cheaper commercial types of inductor is often as high as 2.10^{-4}/°C and is somewhat indeterminate. Moreover, the resistive components of the impedances of the inductors play a considerable part in the performance of these discriminators.

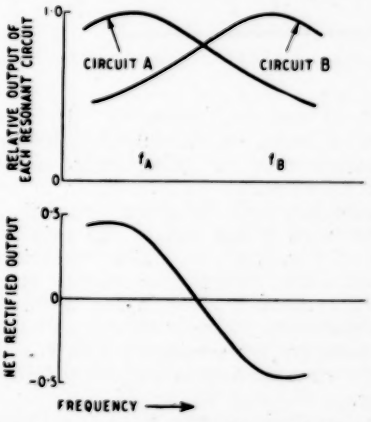


Fig. 1. *Characteristics of the discriminator based on two resonant circuits with staggered frequencies of resonance.*

That fraction of this component which is due to the copper losses has a high temperature coefficient, though the degradation of "Q," which is in some cases permissible and in others desirable, can be carried out with materials having a small and, to some extent, compensating temperature coefficient of resistance. Where the frequencies involved are low, say below 10 kc/s, iron-cored inductors are usually necessary; a new factor, the non-linearity of the in-

* MS accepted by the Editor, February 1946.

ductance of such components, must now be considered unless the signal level is kept very low.

A need thus exists for a discriminator which does not require the use of inductors.

Resistance-Capacitance Discriminator

Whereas the performance of the simple discriminator mentioned above is based on the shape of the response curve of a tuned circuit at and near resonance, the present design[2] is based on the output v. frequency characteristic of the Wien bridge near the balance frequency. In this paper, however, consideration will be given primarily not to the Wien bridge but to the twin-T network of Fig. 2 (a), to which the bridge is related. The bridge equivalent of this twin-T network is given in Fig. 2 (b). This more elaborate bridge or the twin-T network can have greater sensitivity than the Wien bridge when used as a frequency bridge or meter.

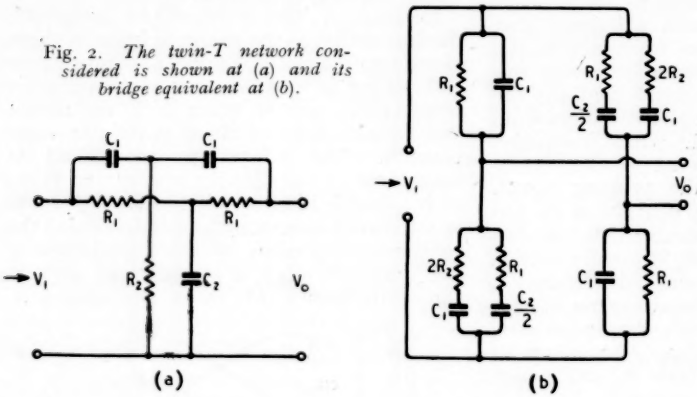


Fig. 2. *The twin-T network considered is shown at (a) and its bridge equivalent at (b).*

The analysis of the twin-T network is simple and need not be given in full here. The conditions for balance, i.e. $V_0 = 0$ for a finite value of V_1 at a frequency $f_0 \left(= \frac{\omega_0}{2\pi}\right)$, are given by the relationships

$$2R_1R_2\omega_0^2C_1^2 = 1 \quad \text{and} \quad \omega_0^2C_1C_2R_1^2 = 2.$$

If $R_1 = nR_2$, $C_1 = \frac{n}{4}C_2$. Now in practice the generator feeding the network may well have a low value of internal impedance and the detector or output voltmeter a high value. Under such conditions the output voltage at a frequency $f \left(= \frac{\omega}{2\pi}\right)$ is proportional to $\frac{\sqrt{n}}{(n+2)}$, which has a maximum value when $n = 2$*. Adopting this value of n, $R_1 = 2R_2$, $C_2 = 2C_1$ and the out-of-balance voltage V_0 at this frequency f is $\frac{(x^2-1)V_1}{(x^2-1)-4jx}$ where $x = f/f_0$. The modulus of this out-of-balance voltage is $+\sqrt{\frac{(x^2-1)^2V_1^2}{x^4+14x^2+1}}$. $V_0(x)$, $[= V_0(1/x)]$, has been evaluated (see Table) for some values of x from 0.6 to 1.66.

When $n = 2$, the four-element arms of the bridge of Fig. 2 reduce to two-element arms as shown in Fig. 3.

Electrical Properties of the Discriminator

The discriminator consists of two such twin-T networks having approximately equal values of R_1 and suitably staggered balance frequencies. The two outputs obtained after rectification, and separate and equal amplification if necessary, are qualitatively as shown in Fig. 4. Here f_1 is the centre

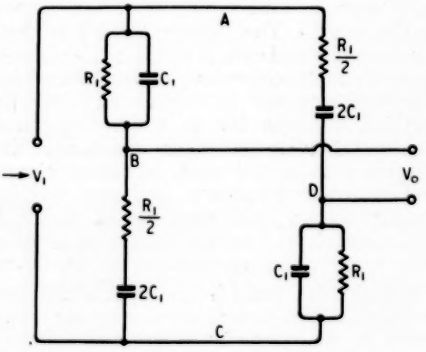


Fig. 3. *When $n = 2$, the bridge of Fig. 2 (b) reduces to this form.*

* For other conditions 2 may still be the value of n which gives a maximum value of V_0, but the expressions now quoted for V_0 no longer hold exactly.

frequency and f_1/m_0 and $m_0 f_1$ are the balance frequencies of the two networks, so chosen that the range between them is the required overall frequency band. If these outputs are placed in opposition the relationship between the differential rectified voltage, V_{DR}, and frequency is given by the full lines of Figs. 5 and 6. The scale of ordinates

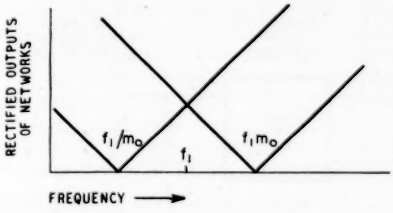


Fig. 4 (above). *Characteristics, in the form of the rectified outputs* v. *frequency, of the two twin-T networks with staggered balanced frequencies.*

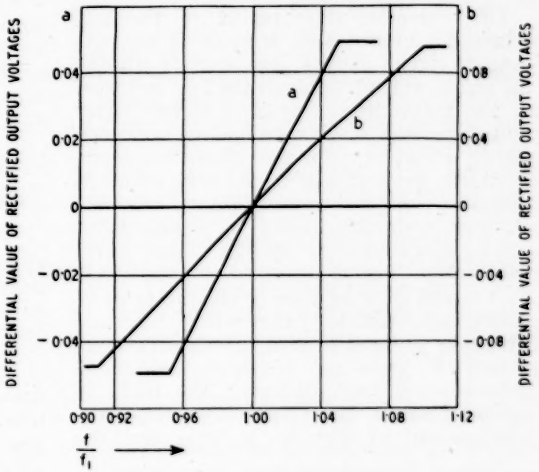


Fig. 5 (right). *Output voltage* v. *frequency, response of discriminator having* (*a*) $m_0 = 1.05$, (*b*) $m_0 = 1.10$.

is in terms of V_1 and assumes both that an input of V_0 volts a.c. to either rectifier produces an output of V_0 volts d.c., and unity amplification. These curves express the relationship

$$V_{DR} = \left| V_0\left(\frac{m_0 f}{f_1}\right)\right| - \left| V_0\left(\frac{f}{m_0 f_1}\right)\right| .$$

The values of m_0 shown were selected arbitrarily; their range covers most applications. The sensitivity in terms of volts output per volt input is approximately $1/f_1$ per c/s; it is nearly independent of m_0. An inspection of the curves of Fig. 4 in the regions of $f > m_0 f_1$ and $f < f_1/m_0$ will provide the reasons for the substantial constancy of V_{DR} outside the important band.

The input impedance Z of a twin-T network, for which $n = 2$, at a frequency f is given by the expression

$$\frac{x(x^2+3) - j(3x^2+1)}{2x(x^2+1)} R_2.$$

For $x = 1$ this reduces to $R_2 - jR_2$. For $x = 2/3$, $Z = (1.19 - 1.21\,j)\,R_2$ and for $x = 1.5$, $Z = (0.81 - 0.79j)R_2$.

Thus Z varies only slowly with x, and the two twin-T networks of a discriminator connected, as they should be, in parallel and having a common value of R_2 present an impedance which changes only very slightly over practical ranges of f_1/m_0 to $f_1 m_0$. If for any reason the reactive component of

VALUES OF $V_0(x) = V_0(1/x)$

x	0.60	0.62	0.64	0.66	0.68	0.70	0.72	0.74	0.76	0.78	0.80
$1/x$	1.667	1.613	1.563	1.515	1.471	1.429	1.389	1.351	1.316	1.282	1.25
V_0/V_1	0.258	0.241	0.225	0.209	0.194	0.179	0.165	0.151	0.138	0.125	0.112

x	0.82	0.84	0.86	0.88	0.90	0.92	0.94	0.96	0.98	1.00	
$1/x$	1.220	1.191	1.163	1.136	1.111	1.087	1.064	1.042	1.020	1.00	
V_0/V_1	0.099	0.087	0.075	0.064	0.053	0.042	0.031	0.020	0.010	0	

the impedance presented to the source of power must be kept small, the value of the inductance of the primary winding of the transformer or of the choke coupling the source to the load should so be chosen as to effect considerable neutralization of this otherwise large capacitive component.

A Refinement

The universal requirement of the discriminator of any f.m. receiver is that its output signal should reproduce faithfully the signal which modulates the f.m. oscillator of the transmitting station. If this latter unit generates an instantaneous frequency whose deviation from the centre frequency is strictly proportional to the instantaneous magnitude of the applied signal, the discriminator should, ideally, give an output whose magnitude is strictly linear with frequency. The characteristics represented by the full lines of Figs. 5 and 6 are more nearly linear with log f than with f, a feature which is perhaps more evident with large values of $(m_0 - 1)$. If simple additional networks are used, however, the linearity against f can be vastly improved. When the arrangement of Fig. 7 is used and the

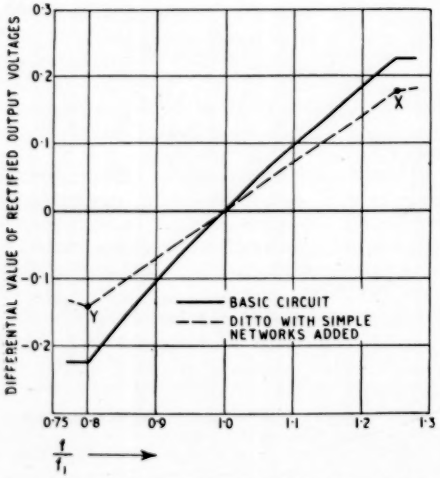


Fig. 6. *Response of discriminator having* $m_0 = 1.25$.

relationships $R_3 \gg R_1$ (of the main networks) and $2\pi f_1 R_3 C_3 = 1$ are satisfied, the broken curve of Fig. 6 can be obtained. The section of this curve between X and Y nowhere departs from the straight line joining X and Y by more than 0.4 per cent of the vertical ordinate at X. It will be seen that the sensitivity is reduced by about 30 per cent for this value, 0.25, of $(m_0 - 1)$. It is not

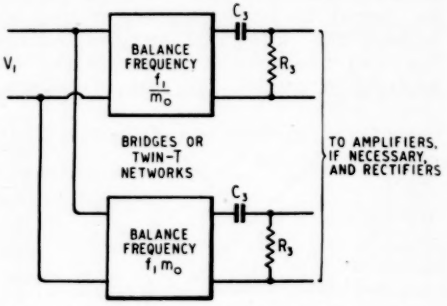


Fig. 7. *The linearity is improved by using the additional networks* R_3C_3.

apparent from a casual inspection of the relationship which this curve represents, namely

$$V_{DR} = \frac{|V_0\,(m_0 f/f_1)| - |V_0(f/m_0 f_1)|}{\{1 + (f_1/f)^2\}^{\frac{1}{2}}},$$

that such a high degree of linearity will result. But if, in the expression for V_0, $4.05\ x$ is substituted for $\sqrt{x^4 + 14x^2 + 1}$ which it equals to within ± 1.2 per cent over the range $x = 0.64$ to 1.56 concerned here, the expression

$$4.05\ V_{DR} = \frac{\left\{\dfrac{(m_0 f/f_1)^2 - 1}{m_0 f/f_1} - \dfrac{1 - (f/m_0 f_1)^2}{f/m_0 f_1}\right\}}{\{1 + (f_1/f)^2\}^{\frac{1}{2}}} V_1$$

is obtained. From this it can be deduced that

$$V_{DR} \propto (f/f_1 - 1)\sqrt{\frac{f/f_1 + \frac{1}{4}(f/f_1 - 1)^2}{f/f_1 + \frac{1}{2}(f/f_1 - 1)^2}}.$$

The term involving the square-root sign differs from unity by less than 0.7 per cent for any value of f/f_1 between 0.8 and 1.25.

The same high degree of linearity results from the use of one such additional network suitably fitted to the input circuit common to the two main networks. High linearity can also be obtained if one twin-T network is followed by a CR network (connected similarly to Fig. 7), for which now $2\pi f_1 C_3 R_3$ should be $\ll 1$, and the other twin-T network by a high value resistive potentiometer which reduces the output at all relevant frequencies, in the ratio $1 : 2\pi f_1 C_3 R_3$. The sensitivity is clearly much reduced in this case, however.

If the relationship between the instantaneous frequency of the f.m. oscillator and the input level is other than a linear one, the departure from linearity being known, high fidelity may still be possible with the use of other correcting networks.

Practical Details

The RC discriminator presents few problems in practice. The resistive components of the impedance of the capacitors and the reactive components of that of the resistors are normally negligible. Stray capacitances from the junction of C_1 and R_2 to earth (see Fig. 2 (a)) should be kept to a minimum : their presence involves slight changes in the values of R_2 and C_2 required to ensure balance at the calculated frequency f_1/m_0 or $m_0 f_1$ and modifies slightly the relationships connecting V_0 and f. The values of R_2 and C_2 should be controllable over small ranges.

Maintenance is simple. Initially and at long intervals R_2 and C_2 of each network should be adjusted so that zero output is obtained from the network concerned at the appropriate balance frequency. The gain of the amplifier following one network should then be adjusted to make V_{DR} zero at f_1. For normal maintenance this latter adjustment is all that is needed. For, besides dealing with changes of amplifier gain, it can be used to counteract any small change of the frequency at which $V_{DR} = 0$ resulting from small changes of either balance frequency. The change of the slope of the characteristic which accompanies this counteraction is small.

Discussion

The output-frequency characteristic of this discriminator does not show any falling off outside the wanted range ; this deficiency should be rectified by the fitting of a suitable filter network to the input circuit to the discriminator.

The sensitivity is low, particularly for small values of $(m_0 - 1)$; additional amplification will overcome this defect.

To offset these disadvantages the discriminator has the following important advantages. The upper limit of V_1 is set only by the permissible rise of temperature of the resistors R_1 and R_2 or the breakdown voltages of the capacitors C_1 and C_2. Power inputs of 0.1 — 1.0 W, such as are obtainable from small receiving valves, are usually permissible ; with such power inputs the final output V_{DR} may be adequate without the additional amplification suggested in the previous paragraph. Levels of input as high as these are not normally practicable with discriminators using iron-cored inductors ; indeed 1 mW may then be the maximum permissible, consistent with a performance which is independent of level.

The linearity is good over the whole required range of frequency. The use of simple additional networks makes this characteristic excellent even for wide-band units.

There are good prospects of making the performance substantially independent of temperature (T). Because each network consists of several components, there are numerous relationships between the temperature coefficients of the important characteristics of these individual components which will ensure that the balance frequency of such a network is independent of T ; i.e., $\frac{\delta f_0}{\delta T} = 0$. The easiest to aim at is probably

$$\frac{\delta C_1}{\delta T} = \frac{\delta C_2}{\delta T} = -\frac{\delta R_1}{\delta T} = -\frac{\delta R_2}{\delta T}.$$

To-day this can be done to such an extent that the magnitude of $\frac{\delta f_0}{f_0 \cdot \delta T}$ need be $\gtrsim 4.10^{-5}/°C$, without recourse to other than mass produced components. Moreover the physical sizes and the thermal capacities of these components are small.

Nearly all that has been said of the twin-T network with $n = 2$ applies to the equivalent bridge (Fig. 3). The main exception concerns the points of the networks from which stray capacitance to earth should be minimized and stabilized. Referring to Fig. 3, the point D should be earthed and earth capacitance to A kept at a low value. R_1 and C_1 in the arm CD should be made controllable. A transformer having two separate equal secondary windings, each screened and balanced with respect to earth should be used to connect the source of signals to the two bridges involved.

Since analysing and testing these types of discriminator and writing this description, I have noted that the Philco Radio and Television Corporation of the U.S.A. and Albright have patented [3] various discriminators primarily connected with the bridge-T network having for its shunt arm an inductor of high "Q". Such a network has high

sensitivity near its balance frequency. A subsidiary claim covers the network of my Fig. 2 (a), with $n = 2$; its use cannot therefore here be claimed to be novel. The patent specification includes little description of the discriminator using this RC network beyond the statements that the sensitivity is rather low and the components inexpensive.

REFERENCES

[1] Sturley gives some examples in *Journ. I.E.E.*, Vol. 92, p. 197 (1945); Hazeltine Corp. and H. A. Wheeler, Brit. Pat. 507,619; R. O. Carter, Brit. Pat. 570,478.
[2] Brit. Prov. Pat. Spec. 30477/45 and 2800/46.
[3] Brit. Pat. 571,314.

CORRESPONDENCE

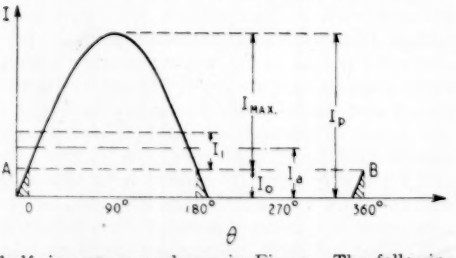
Letters of technical interest are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

Peak Pulse Currents in Class B Amplifiers

To the Editor, "Wireless Engineer"

SIR,—In Class B operation it is often necessary to estimate the peak pulse anode current taken by the valve. If it is assumed that the $I_a V_g$ characteristic of the valve (or valves) is linear and that the latter is biased to zero anode current, the peak pulse anode current is simply π times the direct anode current when sinusoidal excitation is applied to the grid, because the anode current waveform is that of a half-sine wave. This ignores the "standing feed" always present in the practical case. It is, of course, possible to obtain a more accurate result by assuming a "standing feed" and employing Fourier analysis of the resultant wave. A simpler method, only slightly less accurate than the Fourier analysis for normal practical cases, where the peak pulse anode current is much greater than the standing feed, was developed by the writer and it may be of interest to others engaged upon Class B problems.

Let us consider a Class B stage having a valve (or valves; two in push-pull are necessary for a.f. but not for r.f. amplification) with linear $I_a V_g$ characteristics and a standing feed of I_0 amps. The wave shape of the resultant current pulse for sinusoidal excitation is slightly greater than a



half-sine wave as shown in Fig. 1. The following components of the anode current wave are important:

I_{max} = peak value of the current pulse above the standing feed value I_0; i.e., the peak value of the half-sine wave part of the current pulse.

I_a = direct current taken by the valve when supplying the current pulse.

I_1 = direct current corresponding to the half-sine wave of current above AB.

I_p = peak pulse current value $= I_0 + I_{max}$.

Since the wave above AB is a half-sine wave

$$I_1 = \frac{I_{max}}{\pi}$$

If the shaded area outside the 0° and 180° positions of the wave is neglected (this assumes that I_p is large compared with I_0) the d.c. component contributed over a complete cycle by the standing feed is $\frac{1}{2}I_0$, and the total current $I_a = I_1 + \frac{1}{2}I_0$

$$= \frac{I_{max}}{\pi} + \tfrac{1}{2} I_0$$

or $I_{max} = \pi(I_a - 0.5\, I_0)$

The peak value of the anode current pulse

$$I_p = \pi(I_a - 0.5\, I_0) + I_0$$
$$= 3.14\, I_a - 0.57\, I_0.$$

Evesham. K. R. STURLEY.

Time-base Converter and Frequency-divider

To the Editor, "Wireless Engineer"

SIR,—In his letter published in the May 1946 issue of *Wireless Engineer*, Dr. Moss calls for an explanation of what he is pleased to describe as the discrepancy between theoretical reasoning and the results shown in the photographs published by us indicating the high degree of linearity obtainable by using the signal converter as the time-base valve in an oscillograph.

The explanation is that there is, in fact, no such discrepancy, except in so far as Dr. Moss made an error in his theoretical reasoning. It is true that, if a resistance of ten megohms is placed parallel to the deflector plates of the c.r. tube and the capacitor across which the time-base potentials are produced, then, assuming a constant charging current (signal-converter output-electrode current) of 50 μA, and a time-base output potential of 200 volts, the linearity error is 20 per cent.

However, the internal "leak" of a c.r. tube is not a pure ohmic load shunting the deflector plates. Dr. Moss makes a mistake when he thus refers to the deflector-plate characteristic in *Wireless Engineer*, August 1941, Fig. 1, because this characteristic shows that the leak due to secondary electrons increases relatively slowly when the potential of the deflector plate is increased above +10 volts relative to the final-anode potential. Thus, although the effective impedance of the tube for which these characteristics are given is less than 10 megohms when the deflector plate is 10 volts positive to final anode, it is evidently greater than 10 megohms when the deflector plate is 100 volts positive.

Now linearity of a time base means the constancy of the time-base velocity, which is proportional to the charging current. Thus, if the charging current is, say, 50 μA, and the "leak" is 1 μA, the linearity error is 2 per cent. The tube for which the characteristics are given in Fig. 1 of the article by Dr. Moss in the August 1941 issue is much inferior to the tubes which are obtainable at the present time. We are employing in our oscillograph a commercial tube (VCR 112 or CV 1112) wherein no special screens or other precautions are employed for the collection of secondary or stray primary electrons. Nevertheless, we found that the current collected by the X-deflector plates (nearest the fluorescent screen), even when 100 volts positive relative to final anode, is less than 1 per cent of the beam current, or about 0.25 μA, when the beam current is 30 μA. This gives a maximum linearity error of 0.5 per cent in our time base due to this cause.

Also a signal converter negative output electrode current of 50 μA is the lowest limit: at the usual beam current value, 0.5 mA, the error of linearity is only 0.05 per cent.

Our calculations in Appendix I in the September 1945 issue of *Wireless Engineer*, naturally refer only to leaks which have an ohmic character.

The false conception of Dr. Moss is mainly due to the blind acceptance of the "axiom of cathode-ray tube work that the deflector-plate driving-source impedance should be kept low." This is an established principle (but by no means a self-evident truth) because, when conventional valves are used, it is not possible to eliminate ohmic resistances of the order of 1 megohm shunting the time-base capacitor.

The necessity for a low-impedance time-base generator input is very real in the case of the split-deflector plate double-beam c.r. tube to which Dr. Moss refers in the latter part of his August 1941 article, as the leakages are considerable due to the interception of the primaries by the splitter plate. A superior solution is the employment of two separate gun systems, as the interaction of the two deflection systems is hardly avoidable in the split type, unless the parallel impedances are very low, with proportional increase of the wattage requirements.

The fact that the load of the signal converter can be wholly reactive is a major advantage, not a "drawback." As we were at pains to explain in our previous publications, the ideal time base *requires* a pure reactive load; only then can true linearity be achieved at high time-base frequencies. This is why the 200 kc/s time-base trace, reproduced in Dr. Moss's letter, shows a linearity error of 30 per cent.

Regarding our previous statement that "When the frequency is such that the capacitances are of the order of the resistances, synchronization is unreliable and the linearity of the scan is destroyed," we agree that the word "synchronization" was inaccurate. We had the "semi-direct driven" time base in mind, which we employ in the oscillograph described in the September and October 1945 issues of *Wireless Engineer*.

Our previous statement that "the signal converter is superior to conventional valves because two parameters are available—the intensity of the beam current and the position of the beam," Dr. Moss considers a doubtful advantage. Surely, if there is no interaction between the various networks which control the modulation of the beam intensity and the various deflector plates, the greater range of application is an advantage without drawbacks. In the conventional intensity-modulated valve, interaction between the control grids is unavoidable; this is not the case in a deflection-modulated cathode-ray valve.

It is not for us to state whether we have "succeeded in establishing any great superiority in time-base technique as a result of the use of the signal converter." But we should like to say that some impartial opinions are more encouraging than those of Dr. Moss.

We quite agree that the huge list of valve types at present in use should be reduced. Ultimately, perhaps, a score of intensity-modulated valves, a score of deflection-modulated types, and some velocity-modulation types, etc., should satisfy the optimum requirements of the circuit engineer.

London, W.1. P. NAGY
M. J. GODDARD

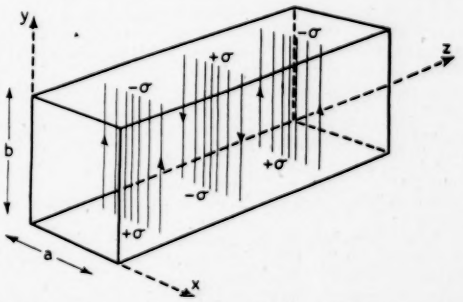
Anomalous Attenuation in Waveguides

To the Editor, The Wireless Engineer

SIR,—In reading Mr. Kemp's interesting paper in your August issue, I was struck by a passage which, if torn from its context, might appear very anomalous. It is on p. 213:—

"The first term is due to currents induced in the top and bottom walls by the tangential magnetic intensities H_x and H_z. . . . currents induced in the side walls. These are due to the longitudinal magnetic field H_z only; for H_x, being perpendicular to the side walls, is prevented from setting up currents in them."

Now all our experience of purely magnetic fields (e.g., the consideration of eddy-currents in the core of an iron-cored transformer) is exactly the reverse of the above, namely that the current in a conductor is proportional to the component of magnetic flux perpendicular to the plane in which the current flows, and some explanation is therefore needed of the different phenomenon occurring in the wave-



guide. The easy answer is to refer to the equations of the *electromagnetic field* (not to components of magnetic intensity) and correct mathematics will then give the correct answer. But there are advantages in a "pictorial" type of presentation, and the writer would therefore like to ask whether there are any fallacies in the following qualitative explanation, or alternatively whether this method can be worked out quantitatively.

First, we can dispose of the magnetic field perpendicular to the side walls by pointing out that at the boundary surface the field is always tangential: at the cut-off frequency (Fig. 6 of the paper) this is obviously so, and at higher frequencies (Fig. 5) the component H_z curves round and becomes H_z before reaching the boundary surface. Then we proceed to deduce the electric currents in terms of the *electric* component of the electromagnetic field. The lines of electric intensity, which form "cores" to the rings of magnetic flux, will in the mode under consideration form a pattern of the type shown in the accompanying figure. (The electric field does, of course, extend over appreciable distances along the Z axis, but for the sake of clarity only sections through the planes of maximum electric intensity have been drawn in the figure.) These lines of electric intensity must terminate, both at the top and the bottom, in surface charges $\pm\sigma$ on the boundary conductors, and as the field reverses cyclically so these charges must reverse in polarity. Clearly this can be effected by interchange of charges both between adjacent positive and negative areas on the same surface and between positive and negative areas on the two boundaries at opposite ends of a given group of lines of electric intensity. The first interchange requires currents in the top and bottom walls only, and the second currents in the side walls, each with accompanying energy dissipation.

The magnitude of the surface charges depends only on the magnitude of the component E_y, but the current involved in the movement of these charges is inversely proportional to the time available for the interchange ($i = dq/dt$) and therefore directly proportional to the frequency; and on this account the dissipation, i^2R, would tend to increase with the square of the frequency. However, at frequencies far above the cut-off frequency the wavelength is approximately inversely proportional to the frequency, so that the separation between adjacent charges of opposite sign (and therefore the path of the current) in the top and bottom walls is decreasing with increasing frequency. Hence the component of energy loss due to eddy currents in the top and bottom walls is, so far, increasing less rapidly than the square of the frequency. But in the meantime, the shortening of the current paths in the top and bottom walls will tend to cause a greater proportion of the interchange of charges to take place along the top and bottom, and less along the side walls in which the path length is independent of frequency. Hence the losses in the side walls will decrease with rising frequency, although in a guide of fixed dimensions the rate of increase of total attenuation is of the order of proportionality to the square of the frequency.

It follows that at infinite frequency all the losses would be in the top and bottom walls, but they would be eliminated if one could avoid terminating the component E_y in surface charges on a conducting boundary. The obvious method of doing this is to extend the dimension b very greatly, and then curve the cross-section into an annulus so that the original E_y becomes a continuous circumferential component, and one can then derive the circular guide by letting the inner radius of the annulus tend to zero.

D. A. BELL

London, N.21.

Pulse Modulation

To the Editor, "Wireless Engineer."

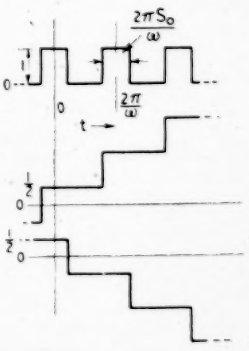
SIR,—In spite of the discussion on this subject in your correspondence columns since the article by Roberts and Simmonds in November 1945, there appears to be still some doubt as to the formulae for the various types of modulated pulse waveforms. Apart from the confusion of definitions for the different types of modulation, there are discrepancies between the "exact" formulae produced by different writers.

The purpose of this note is to point out that all types of modulated pulse trains with constant amplitude pulses can be represented by a single general formula, from which formulae for particular types of modulation can easily be derived as special cases.

Modulated pulse trains differ from unmodulated trains in the positions, in time, of the leading or trailing edges, or both, of the pulses. Usually the time displacements of the leading and trailing edges are related in a simple way, but it is convenient to consider the general case in which the modulations of leading and trailing edges are unrelated.

Any pulse train, modulated or unmodulated, consisting of constant amplitude pulses with vertical sides (it is convenient to take the amplitude as unity), may be regarded as the sum of two stepped waves or staircases, the first of which is a positive or ascending staircase, with positive steps of unit height occurring at the positions of the leading edges of the pulses; while the other staircase has negative steps, also of unit height occurring at the positions of the trailing edges of the pulses.

Consider an unmodulated pulse train with a repetition frequency $\omega/2\pi$ per second, and pulse width $2\pi S_0/\omega$. The zero of the time scale, which is arbitrary, may be taken at the centre of a pulse. The ascending staircase corresponding to the leading edges may have the arbitrary value $\frac{1}{2}$ at $t = 0$ and increase by 1 at the times $2\pi/\omega - \pi S_0/\omega$, $4\pi/\omega - \pi S_0/\omega$, etc., remaining constant between steps, while the descending staircase, corresponding to the trailing edges, is equal to $\frac{1}{2}$ at $t = 0$ and decreases by 1 at the times $\pi S_0/\omega$, $2\pi/\omega + \pi S_0/\omega$, etc. It is easily shown that the equation to the ascending staircase is



$$\frac{S_0}{2} + \frac{\omega t}{2\pi} + \sum_{k=1}^{\infty} \frac{1}{\pi k} \sin k(\omega t + \pi S_0) \quad \ldots \quad (1)$$

Suppose now that the positions of the leading edges, and thus of the staircase steps, are modulated by a signal $f_1(t)$ in such a way that the time displacements of the edges from their unmodulated positions are proportional to the values of $f_1(t)$ at the times at which the steps take place; e.g., a

step which in the absence of modulation occurs at time t_1 now occurs at $t = t_1 - \delta t$, where $\delta t = \phi_{max} f_1(t)/\omega$, ϕ_{max} being the phase deviation and ϕ_{max}/ω the time deviation when $f_1(t) = 1$. The equation to the modulated staircase is then obtained by writing $t + \phi_{max} f_1(t)/\omega$ in place of t in expression (1), and it can be seen that this transformation satisfies the condition for the position of the pulse edges, laid down in the previous sentence, at every step in the staircase, and also, of course, gives a constant value to the staircase between steps.

The equation to the unmodulated descending staircase is

$$\frac{S_0}{2} - \frac{\omega t}{2\pi} - \sum_{k=1}^{\infty} \frac{1}{\pi k} \sin k(\omega t - \pi S_0)$$

If the trailing edges are modulated by a different signal $f_2(t)$ so that the time displacements are equal to $\phi_{max} f_2(t)/\omega$, then the equation to the modulated staircase is obtained by writing $t + \phi_{max} f_2(t)/\omega$ for t. The equation to the complete pulse train, obtained by adding the expressions for the two modulated staircases, is

$$S_0[1 + A\{f_1(t) - f_2(t)\}/2] + \sum_{k=1}^{\infty} \frac{2}{\pi k} \sin k\pi S_0 [1 + A\{f_1(t) - f_2(t)\}/2] \cos k[\omega t + \pi A S_0 \{f_1(t) + f_2(t)\}/2]$$

in which $A = \phi_{max}/\pi S_0$. . . . . (2)

From this general formula the equation for any particular type of pulse modulation may be obtained by assigning appropriate values to $f_1(t)$ and $f_2(t)$.

For the type of "symmetrical" pulse-length modulation generated by the usual sliced sawtooth type of modulator $f_2(t) = -f_1(t)$, and expression (2) becomes identical with equation (6) of Roberts and Simmonds when $f_1(t) = \cos pt$.

When the phase of the repetition frequency generator is modulated, $f_2(t) = f_1(t)$ and if $f_1(t) = \sin pt$ expression (2) becomes equal to equation (7) of Roberts and Simmonds, which, however these authors describe as "delay modulation."

For modulation of the leading pulse edges only, $f_2(t) = 0$, while for modulation of the trailing edges only, $f_1(t) = 0$. In the general case of a symmetrical pulse-length modulation $f_2(t) = -bf_1(t)$ where b is a positive constant.

An important case is that in which the time duration of each pulse is constant and equal to the unmodulated value $2\pi S_0/\omega$, and the position of the pulse is varied by the modulating signal (pulse-position modulation). In modulators producing this type of pulse train, the position of the leading edge is determined by the modulating signal, and the trailing edge is generated automatically therefrom by a pulse forming network giving a constant time delay. The condition of constant pulse length is obtained by writing

$$f_2(t) = f_1(t - 2\pi S_0/\omega)$$

A more convenient expression is obtained by assuming that the positions of the pulse centres are modulated by $f(t)$ and that

$$f_1(t) = f(t + \pi S_0/\omega) \qquad f_2(t) = f(t - \pi S_0/\omega)$$

This is equivalent to the trivial change of imposing a time advance of $\pi S_0/\omega$ on the modulating wave.

On substituting these values for $f_1(t)$ and $f_2(t)$ in the general formula, and writing $f(t) = \sin pt$, the expression for the pulse train becomes, after expansion in terms of Bessel functions,

$$S_0 + \frac{\phi_{max}}{\pi} \sin(\pi S_0 p/\omega) \cos pt + \cdot \sum_{k=1}^{\infty} \sum_{n=-\infty}^{\infty} \frac{2}{\pi k} J_n(k\phi_{max}) \sin \pi S_0 (k + np/\omega) \cos(k\omega + np)t$$

In this particular example the calculation is most conveniently carried out by resolving the general formula back into its two staircase components.

On comparing this expression with that given by Mr. Fitch in your August issue, there is seen to be agreement between the first two terms, but a considerable discrepancy in the last group of terms. It is difficult to say at what point the difference arises, since Mr. Fitch does not explain how his formula is derived or on what assumptions the derivation is based. No doubt the matter will become clearer when Mr. Fitch's full analysis is published.

Research Laboratories of the General Electric Co., Ltd., Wembley. A. S. GLADWIN

[Mr. Gladwin asks us to add that since the above was written his attention has been drawn to reports by Dr. A. Bloch on various types of pulse modulation in which a formula similar to (2) was deduced by another method, which also used the argument of a transformation of the time scale.—ED.]

To the Editor, "Wireless Engineer."

SIR,—In reviewing my letter in the August issue, I noticed the following omission:

The letter ν should multiply the quantity πS in the last two equations given in the appendix. These equations should therefore read

$$e = S + 2/\pi \sum_{\nu=1}^{\infty} \left[1/\nu \sin(\nu\pi S)\right] \cos \nu\omega t$$

and

$$e(S, \theta) = S + 2/\pi \sum_{\nu=1}^{\infty} \left[1/\nu \sin(\nu\pi S)\right] \cos \nu\theta.$$

Although this error is unfortunate, it does not affect the arguments or results of my previous communication in any way.

E. R. KRETZMER.
Massachusetts Institute of Technology, Cambridge, Mass., U.S.A.

BOOKS RECEIVED

An Index of Mathematical Tables by A. Fletcher, M.A., Ph.D., J. C. P. Miller, M.A., Ph.D., and L. Rosenhead, Ph.D., D.Sc. Published by Scientific Computing Service Ltd., 23, Bedford Sq., London, W.C.1. Pp. 450 + viii. Price 75s.

CORRECTION

In "Anomalous Attenuation in Waveguides," which appeared in *Wireless Engineer* for August 1946, p. 211, the drawings of Figs. 8 and 11 were interchanged. The captions are correct for the figure numbers, but not for the drawings above them. In addition, the symbol θ on both drawings should be ϕ.

WIRELESS PATENTS

A Summary of Recently Accepted Specifications

The following abstracts are prepared, with the permission of the Controller of H.M. Stationery Office, from Specifications obtainable at the Patent Office, 25, Southampton Buildings, London, W.C.2 price 1/- each.

ACOUSTICS AND AUDIO-FREQUENCY CIRCUITS AND APPARATUS

576 197.—Arrangement of spaced microphones for indicating the level of volume and reverberation in an auditorium, particularly for monitoring the reproduction of sound films.

H. W. Hastings-Hodgkin and Tickford Ltd. Application date 25th April, 1944.

576 322.—Flexible rod-aerial for a motor-car, arranged to be operative either when the free end is erect, or when it is anchored to lie parallel with the roof of the car.

A. F. Burgess (communicated by Zenith Radio Corp.). Application date 21st February, 1944.

576 344.—Circuit for detecting the presence of conducting bodies in an opaque mass of insulation, or any asymmetry of the coaxial wire relatively to its sheath in a cable.

Marconi Instruments Ltd., W. B. Bartley and C. F. Brockelsby. Application date 17th January, 1944.

576 349.—Magnetic system forming a gap of inverted-T shape for minimizing distortion in a gramophone pick-up of the moving-coil type.

Radio Gramophone Development Co. Ltd. and S. T. Bolus. Application date 7th February, 1944.

AERIAL AND AERIAL SYSTEMS

576 159.—Directive aerial of the helical type, wherein the wire forming the larger helix is first wound into a spiral of smaller diameter and pitch [addition to 573 896].

Marconi's W.T. Co. Ltd. and C. S. Franklin. Application date 4th December, 1942.

576 360.—Highly-aperiodic short-wave aerial comprising two parallel strips of metal, connected to form a narrow loop by parabolic end-pieces, the feed-line being connected to a centre gap in one of the strips.

Soc. Anonyme des Industries Radio-Electriques. Convention date (France) 28th March, 1941.

RECEIVING CIRCUITS AND APPARATUS

(See also under Television)

576 103.—A.V.C. circuit, utilizing an amplifier as an impedance-transformer, in a receiver for pulsed signals, as used in radiolocation.

The General Electric Co. Ltd. and E. C. Cherry. Application date 18th December, 1942.

576 154.—Selecting and then permanently holding the sensitive point-contact between a catswhisker and a rectifying crystal for very-short waves.

Sperry Gyroscope Co. Inc. (assignees of F. S. Salisbury). Convention date (U.S.A.) 19th May, 1941.

576 158.—Time-base circuit for a C.R. tube designed to show two discrete traces against scanning-sweeps of different speeds.

Cinema-Television Ltd. and S. S. West. Application date 17th November, 1942.

TELEVISION CIRCUITS AND APPARATUS

FOR TRANSMISSION AND RECEPTION.

576 204.—Cathode-ray tube designed to facilitate the viewing of a televised picture at an angle to the fluorescent screen and to the main axis of the scanning stream.

Cinema-Television Ltd. and T. M. C. Lance. Application date 26th April, 1944.

576 277.—Amplifier with negative feedback for offsetting the attenuation of the higher frequencies when receiving a wide band of signals as in television.

Marconi's W.T. Co. Ltd. (assignees of O. H. Schade). Convention date (U.S.A.) 20th January, 1943.

TRANSMITTING CIRCUITS AND APPARATUS

576 145.—Variable-attenuating device for a waveguide, consisting of a resistive septum or plate adjustably spaced from one of the longitudinal walls of the guide.

Western Electric Co. Inc. Convention date (U.S.A.) 7th May, 1943.

576 147.—Conical coupling-device for connecting two coaxial cables, of unequal outer diameters, transmitting waves of the order of 10 cm.

The General Electric Co. Ltd. and R. J. Clayton. Application date 19th April, 1941.

576 152.—Symmetrical "neutrodyning" arrangements for the electrodes and mountings of high-powered short-wave amplifiers.

Soc. Française Radio-Electrique. Convention date (France) 27th February, 1941.

CONSTRUCTION OF ELECTRONIC-DISCHARGE DEVICES

576 003.—Spacing and construction of the electrostatic lens-assembly of an electron microscope.

The British Thomson-Houston Co. Ltd. Convention date (U.S.A.) 1st December, 1942.

576 085.—Screw-thread type of attachment between the re-entrant glass tube and the electrode system of a thermionic valve.

The M-O. Valve Co. Ltd. and L. R. E. Windsor. Application date 12th January, 1942.

576 096.—Magnetron oscillator in which permanent magnets are arranged to allow for varying the intensity and direction of the control-field.

The General Electric Co. Ltd. and E. M. Hickin. Application date 21st September, 1942.

576 121.—Regulating the drift-time of the electrons in a velocity-modulation tube for handling centimetre waves.

B. J. Mayo. Application date 8th August, 1941.

576 126.—Arrangement of the hollow resonator and reflecting electrodes in a rhumbatron device for handling centimetre waves.

B. J. Mayo. Application date 8th August, 1941.